不定期偵測下租賃系統之多動作維護策略
A Maintenance Policy with Multiple Choices of Actions for the Leased System under Irregular Inspection

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摘要

本文為租賃系統提出一個多動作維護策略。該策略主要是在每一個偵測時間，依當時之系統狀態，在多動作集中選定最佳維護動作，並選擇下一個偵測點。文中假設：(1) 在每個偵測時間，皆有多個維護動作可供選擇，(2) 每一個選定之維護動作皆為“最小維護”，(3) 偵測時間與維護時間皆可忽略。最後；利用出租影印機為例，說明該租賃系統如何依本策略進行維護。結果顯示，當該系統採用本策略後，其總成本與總操作成本可分別減少 11.5%與 63.6%。
關鍵詞：維護策略，租賃系統，多維護動作，偵測，最小維護

Abstract

A maintenance policy is proposed herein for a leased system under irregular inspection. Such policy is mainly to determine the maintenance action optimally at each inspection time and also to determine the next inspection time based on the current system conditions. Such policy holds by considering that: (1) multiple maintenance actions can be chosen at each inspection time; (2) each maintenance action taken is regarded as a “minimal repair”, and (3) the times spent on inspection and maintenance action are ignored. A numerical example is presented to illustrate how such policy can be carried out for the leased photocopier system in practical. The results illustrate that the total cost and the total operating cost of the leased system under such policy can save 11.5% and 63.6% respectively.
Keywords: maintenance policy, leased system, multiple maintenance actions, inspection, minimal repair

Nomenclature

T_n	the *n*th inspection time at which the *n*th maintenance decision is made
$A=\left\{a^0,a^1,a^2,a^3,a^R\right\}$	the candidate action set at each inspection time T_n, where a^i for $i=1,2,3$ denotes three kinds of maintenance levels, a^0 and a^R denote “*do nothing*” and “*replacement*” respectively
C^p	the penalty cost due to per system failure
$\Lambda(t)$ and $\lambda_0(t)$, $\forall t\geq 0$	the failure intensity functions of the system with\without maintenances respectively
$m(\cdot)$	the scheduling inspection function
N_L^0	the random number of failures produced by the system under no maintenances over $[0,\ L]$
N_L^A and $N_{T_n}^A$	the random number of failures produced by the system with maintenances over $[0,\ L]$ and $[0,T_n]$ respectively
$\Delta_{(T_{n-1},T_n)}(a^k)$	the random number of failures produced over $[T_{n-1},T_n]$ given that a^k is taken at T_{n-1} for $a^k\in A$
$P_{(0,L)}$	the associated penalty costs occur over $[0,\ L]$ while the system is under no maintenances

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M	the planning period for decision use
$P_{(T_n,T_n+M)}\left(a^k\right)$	the associated penalty costs occur over $\left[T_n,T_n+M\right]$ while a^k is taken at T_n
$\tau_{(T_n,T_n+M)}\left(a^k\right)$	the associated total expected cost spent over $\left[T_n,T_n+M\right]$ while a^k is taken at T_n

I. INTRODUCTION

Due to usage and/or ageing, the leased system deteriorates worse when time goes on and ultimately fails [1]. There has been a trend towards leasing equipment rather than owning the equipment since 1990. Such deterioration incurs system failures and the associated penalty costs. As a result, maintenance was no longer an issue for the lessee who leases the equipment, but for the lessor (i.e. the owner of the equipment) who carried out the maintenance. Often, the contract specifies the penalties while the leased equipment doesn't perform as required (for example, failing frequently) or the maintenance service not being satisfactory (for example, repairs are not performed within reasonable times limits) as these affect the business performance of the lessee. Two kinds of maintenances are always taken into consideration for the deterioration systems; (1) through corrective maintenance (CM), the failure system can be restored to an operational one; and (2) through preventive maintenance (PM), the rate of degradation can be controlled. While the level of maintenance effort increases, the penalty costs decrease but the maintenance costs increase. This implies that the optimal maintenance need to be decided by taking the penalty costs into consideration.

Nakagawa [2], Sheu and Chang [3] dealt with sequential PM, and others [4-7] dealt with periodic PM. Park [8] considers a continuous deterioration system which fails when its deterioration level surpasses a breakdown threshold. Under periodic inspections, the policy is mainly to determine the optimal threshold γ^* by minimizing long-run total mean cost per unit time. Accordingly, the system is replaced if the current deterioration level exceeds γ^*, and do nothing if otherwise. Dieulle *et al.* [9] deals with a continuous deterioration system which is inspected at random times chosen by a scheduling function $f(t)$. The policy is to determine the optimal scheduling function $f^*(t)$ and the optimal threshold state S^* to minimize the long-run expected cost per unit time. For current state X_t, the maintenances will be taken according to: (1) if $X_t \geq L$, "*corrective replacement*" is taken, where L is a pre-set failure level, (2) if $L > X_t \geq S^*$, "*preventive replacement*" is taken, (3) otherwise, "*do nothing*". Wang [10] deals with a survey of the maintenance policies for the deteriorating systems.

The maintenance policies are proposed by several researchers [11-14] for the leased system. Such policies consider two types of restoration: (1) failure rate reduction and (2) age reduction. Wang [11] studied "age reduction", and the study "failure rate reduction" [12-14]. Pongpech and Murthy [13] propose a periodic policy where the PM is carried out at periodic times jT over the lease period. T is constant time interval between successive PM actions. Yeh and Chang [14] propose a policy to investigate the optimal threshold value of failure rates for leased products with a Weibull lifetime distribution. Jaturonnatee *et al.* [12] develops a PM policy for a leased system by considering that: (1) each maintenance action taken is "*minimal repair*", (2) the failures produced by the system with/without maintenances occur according to a non-homogeneous Poisson process (NHPP). The policy is to determine the optimal parameters $\left(k,\ \underline{t},\ \underline{\partial}\right)$ by minimizing the expected total cost over the lease period, where (1) k denotes the number of PM actions taken over the lease period, (2) $\underline{t}=\left(t_1,...,t_k\right)$ in which t_k denotes the time instants for the kth PM action, and (3) $\underline{\partial}=\left(\partial_1,...,\partial_k\right)$ in which ∂_k denotes the restored level of the kth PM action.

Three shortcomings of the policy in [12] are listed as follows:

1. The optimization problem involves (2k+1) parameters that need to be selected optimally. It is too complex to be applied in practical.
2. The time intervals between successive PM actions are not constant.
3. Such policy can't reflect the system deterioration in time.

In this study, a maintenance policy is proposed for the leased system presented by Jaturonnatee *et al.* [12]. Such policy is mainly to determine the maintenance action optimally at each inspection time and also to determine the next inspection time. To overcome the shortcomings, the proposed policy is carried out by considering:

1. The optimal maintenance action is selected from the candidate actions set.
2. The inspection times are dynamically updated based on the current system conditions.

That is, only two parameters need to be determined based on the current system conditions at each inspection time.

II. SYSTEM DESCRIPTIONS AND POLICY ASSUMPTIONS

The leased system considered herein satisfies the following characteristics:

1. The system deterioration is time-dependent. That is, such system deteriorates worse while time goes on.
2. The lease contract involves penalty costs for the lessor while the system failures occur during the leased period.

3. Under no maintenance case, the associated penalty costs occur over $[0, L]$ is given by $P_{(0,L)} = C^p \cdot N_L^0$.

4. The failures produced by the leased system with\without maintenances occur both according to the non-homogeneous Poisson process (NHPP). Accordingly, we let $\Lambda(t)$ and $\lambda_0(t)$ $\forall t \geq 0$ denote the failure intensity functions of such system with and without maintenances respectively. In fact, $\lambda_0(t)$ is non-decreasing due to the time-dependent system deterioration.

To carry out the maintenance policy, more assumptions are required as followings:

A1: At T_n for $T_n < L$, one and only one action chosen from the candidate action set $A = \{a^0, a^1, a^2, a^3\ a^R\}$ is taken. While $T_n \geq L$, the system will be replaced and a new leased period begins.

A2: While a^i for $i = 1,2,3$ is taken, the hazard function immediately after a^i being taken is the same as that just before failure [11].

A3: The times spent on inspection and maintenance are both neglected.

A4: State F which denotes the system's sudden and temporary interruption due to fatal shock may also occur. The occurrence of F is self-indicative but the probability that such F occurs at each T_n is ignored. Whenever F occurs, minimal repair is taken immediately in negligible time and costless.

III. THE MAINTENANCE POLICY

For the leased system, the maintenance decision is made at each inspection time T_n according to the following steps:

Step 0: Identify the current system conditions (i.e. the number of the failures produced by the leased system).

Step 1: Determine the optimal maintenance action A_n^*.

Step 2: Determine the next inspection time T_{n+1}.

The details on how to determine A_n^* and T_{n+1} at T_n are illustrated in the following sections.

1. The scheduling inspection function $m(\cdot)$

Due to the time-dependent system deterioration; such leased system should be inspected more frequently when time goes on to identify the current conditions in time. For this purpose, we allow irregular inspection date in this study. At T_n, the next inspection time (i.e. T_{n+1}) can be dynamically updated on the basis of the present conditions which is revealed by inspection only to reflect the system deterioration. The sequence of inspection times $(T_n)_{n=0,1,2,...}$ is strictly increasing and the possibility of an infinite number of inspections occurring on a finite interval is avoided. We let $m(\cdot)$ be the scheduling inspection function which is a decreasing function from $\left[0, N_L^A\right)$ to $[1, m_{Max}]$, where m_{Max} denotes the maximum period that such system must be inspected. In all these cases, such T_{n+1} is chosen by the rule

$$T_{n+1} = T_n + m\left(N_{T_n}^A\right) \tag{1}$$

where $N_{T_n}^A = N_{T_{n-1}}^A + \Delta_{(T_{n-1},T_n)}(a^k)$ in which $\Delta_{(T_{n-1},T_n)}(a^k)$ denotes the random number of failures produced over $[T_{n-1}, T_n]$ given that a^k is taken at T_{n-1} for $a^k \in A$. Such $\Delta_{(T_{n-1},T_n)}(a^k)$ can be identified only by inspection at T_n in neglected time.

2. The determination of A_n^* at T_n

The associated penalty costs occur during $[T_n, T_{n+1}]$ also increase while time goes on due to system deterioration. Thus, the proper maintenance actions should be taken to reduce the number of system failures during the leased period. This decreases the penalty costs but increases the expense of maintenance costs. It implies that the optimal maintenance action taken at T_n needs to be decided through a proper trade-off between penalty and maintenance costs.

Now, we show how to determine the optimal maintenance action A_n^* at T_n as followings. First of all, we need to determine at T_n the function $\Lambda(t)$ which denotes the failure intensity function given that the maintenance actions have been taken at $T_s = t_s$ for $s = 1,...,n\text{-}1$. According to the definition in [11], $\Lambda(t)$ can be determined by

$$\Lambda(t) = \begin{cases} \lambda_0(t) & \text{for } 0 \leq t < t_1 \\ \lambda_1(t) = \lambda_0(t) - \delta_1^k & \text{for } t_1 \leq t < t_2 \\ \ldots & \\ \lambda_{n-1}(t) = \lambda_{n-2}(t) - \delta_{n-1}^k & \text{for } t_{n-2} \leq t < t_{n-1} \\ \lambda_n(t) = \lambda_{n-1}(t) - \delta_n^k & \text{for } T_n \leq t \end{cases} \tag{2}$$

where

(1) $\delta_s^k = \theta^k \cdot \lambda_{s-1}(t_s)$ which denotes the reduction in the intensity function restored by the action a^k taken at $T_s = t_s$ and θ^k denotes the restored factor of the action a^k;

(2) $\lambda_{s-1}(t)$ for $t_{s-1} \leq t < t_s$ denotes the intensity function just before the action a^k being taken at $T_s = t_s$.

Fig.1 illustrates the failure intensity functions of the systems with/without maintenances.

Suppose that the action taken at $T_n = t_n$ is a^k, the associated total expected number of failures produced over the planning period $[t_n, t_n + M]$ can be determined by

$$\begin{aligned} & E(\Delta_{(t_n,t_n+M)}(a^k)) \\ & = \int_{t_n}^{t_n+M} \lambda_n(t)dt \\ & = \int_{t_n}^{t_n+M} \left(\lambda_{n-1}(t) - \theta^k \cdot \lambda_{n-1}(t_n)\right) dt \quad \text{for } k = 0,1,2,3 \end{aligned} \tag{3}$$

where M is a pre-set integer for decision use. While a^R is taken, a new leased period begins.

Let $\tau_{(T_n,T_n+M)}(a^k)$ be the associated total expected cost spent over $[T_n, T_n + M]$ while a^k is taken at T_n. It is noted that

$$\tau_{(T_n,T_n+M)}\left(a^k\right) = \begin{cases} \tau_{(T_n,T_n+M)}\left(a^i\right) = C^0 + C\left(a^i\right) + E\left(P_{(T_n,T_n+M)}\left(a^i\right)\right) & \text{for } i=0,1,2,3 \\ \tau_{(T_n,T_n+M)}\left(a^R\right) = C^0 + C\left(a^R\right) \end{cases} \quad (4)$$

where

(1) C^0 and $C\left(a^k\right)$ denote the costs to take each inspection and a^k respectively.

(2) $E\left(P_{(T_n,T_n+M)}\left(a^k\right)\right) = C^p \cdot E(\Delta_{(T_n,T_n+M)}(a^k))$ in which $P_{(T_n,T_n+M)}\left(a^k\right)$ denotes the associated penalty cost occurs over $[T_n, T_n+M]$ in case that a^k is taken at T_n.

We thus choose the optimal action A_n^* at T_n by

$$A_n^* = \arg \underset{k\in\{0,1,2,3,R\}}{Min}\left\{ \tau_{(T_n,T_n+M)}\left(a^k\right)\right\} \quad (5)$$

IV. ILLUSTRATION

We apply the proposed maintenance policy to the leased photocopier system in practical. The lease contract of such system includes that: (1) the leased period is $L=100(\text{day})$, (2) the lessor carries out the maintenances over the leased period, and (3) while system failures occur, the penal cost for each failure is $C^p = 50(\$)$. We let $\lambda_0(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}$ denote the hazard function of such a system where $\alpha=10$, $\beta=1.8$. We also choose a linear scheduling inspection function by

$$m\left(N_{T_n}^A\right) = 1 + Max\left(\left\lfloor (m_{Max}-1) - \frac{(m_{Max}-1)\cdot N_{T_n}^A}{500}\right\rfloor, 0\right) \quad (6)$$

where $\lfloor x \rfloor = Max\{k \text{ is integer} | k \le x\}$. Other numerical settings are further required as followings: $\theta^0 = 0$, $\theta^1 = 0.2$, $\theta^2 = 0.5$, $\theta^3 = 0.8$, $M=15(\text{day})$, $C^0 = 10(\$)$, $C(a^1) = 300(\$)$, $C(a^2) = 600(\$)$, $C(a^3) = 1200(\$)$ and $C(a^R) = 100000(\$)$.

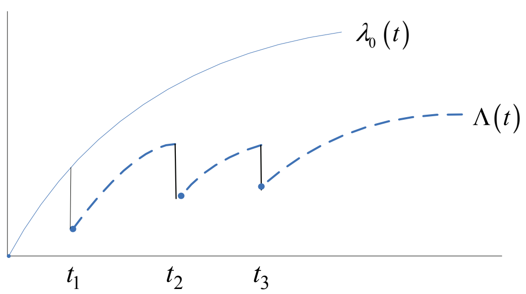

Fig. 1 The failure intensity functions of the systems with/without maintenances

1. The maintenance history of the leased system

The considered system is inspected at T_n for n=1,2,…, and at it we are to determine A_n^* and T_{n+1} respectively till the first integer T so that $T > L$. Such procedure is carried out according to the followings:

(1) Set $T_0 = N_0^A = 0$ and $m_{Max} = T_1 = 10$.

(2) At T_n for $T_n < L$, we are to determine A_n^* and T_{n+1} by applying Eq.(5) and Eq.(1) respectively.

(3) If $A_n^* = a^i$ for $i \in \{0,1,2,3\}$, determine the resulting intensity function by $\lambda_n(t) = \left(\lambda_{n-1}(t) - \theta^i \cdot \lambda_{n-1}(T_n)\right)$ for $t \ge T_n$.

(4) The procedure stops either at $A_n^* = a^R$ or the instant when $T_n \ge L$.

The results are summarized in Table 1. The total operating cost discussed herein includes penalty cost, maintenance cost and inspection cost. The total cost is the sum of total operating cost and replacement cost. Hence, the benefits of such system by taking maintenances can be obtained through comparing such results to that under no maintenances in terms of two indices $R_{top}(C^p, M)$ and $R_{tc}(C^p, M)$ respectively, where

$$R_{top}(C^p, M) = \frac{\text{Total operating cost for the system with maintenance given}(C^p, M)}{\text{Total operating cost for the system without maintenance}} \quad (7)$$

$$R_{tc}(C^p, M) = \frac{\text{Total cost for the system with maintenance given}(C^p, M)}{\text{Total cost for the system without maintenance}} \quad (8)$$

For this purpose, the system under no maintenance is also inspected at each T_n to get the inspection data till the first integer T so that $T > L$. The inspected results are listed in Table 1.

Then, we determine $R_{top}(C^p, M)$ and $R_{tc}(C^p, M)$ by

$$R_{top}(C^p, M) = \frac{3600+120+7642.07}{170+22459.44} 0.374 \quad (9)$$

$$R_{tc}(C^p, M) = \frac{3600+120+7642.07+100000}{170+22459.44+100000} = 0.885 \quad (10)$$

From Eq.(9-10), we observe that the leased system if taking the proposed maintenance policy can save 11.5% in total cost and 63.6% in total operating cost.

We further evaluate such $R_{top}(C^p, M)$ and $R_{tc}(C^p, M)$ for more $M \in \{8,10,18,20\}$, $C^p \in \{30,80\}$ and $\alpha \in \{5,8,15,20\}$ respectively. The results are summarized in Table 2-5. We observe from such Tables that:

(1) For $(\alpha, M) = (10,15)$, the leased system under the policy gets more benefits while C^p gets larger. Table 3 illustrates the maintenance histories of the systems under different C^p.

(2) For $(\alpha, C^p) = (10,50)$, the leased system under the policy save 12.4% in total cost and save 66.9% in total operating cost while we choose M=18.

Table 1 The results of the system under the proposed policy

Policy	Maintenance cost (Mc)	Inspection cost (Ic)	Penalty cost (Pc)	$R_{tc}(50,15)$	$R_{top}(50,15)$	Maintenance time
No maintenance	-	170	22459.44	-	-	NA
Under the policy	3600	120	7642.07	0.885	0.374	20, 29, 47, 56, 73, 89

Table 2 The benefits for the leased system under different C^p

C^p	Policy	The number of Maintenances	Total cost	$R_{tc}(C^p,15)$	Total operating cost	$R_{top}(C^p,15)$
30	No maintenance	0	113646	-	13646	-
	Under the policy	3	107760	0.948	7760	0.569
50	No maintenance	0	122629	-	22629	-
	Under the policy	6	108468	0.885	8468	0.374
80	No maintenance	0	131613	-	31613	-
	Under the policy	7	109716.14	0.834	9716.14	0.307

Table 3 The maintenance histories of the systems under different C^p

C^p=30			C^p=50			C^p=80		
No.	Maintenance time	Optimal action	No.	Maintenance time	Optimal action	No.	Maintenance time	Optimal action
1	38	a^2	1	20	a^2	1	20	a^2
2	54	a^2	2	29	a^2	2	30	a^2
3	77	a^2	3	47	a^2	3	39	a^2
4	100	*replacement*	4	56	a^2	4	57	a^2
			5	73	a^2	5	65	a^2
			6	89	a^2	6	73	a^2
			7	100	*replacement*	7	88	a^2
						8	95	a^2
						9	100	*replacement*

Table 4 The benefits for the system under different M

M	The number of Maintenances	Total cost	$R_{tc}(50,M)$	Total operating cost	$R_{top}(50,M)$
8	3	113864	0.929	13846	0.613
10	4	110849	0.904	10849	0.479
15	6	108468	0.885	8468	0.374
18	8	107481	0.876	7481	0.331
20	7	109838	0.896	9838	0.435
No maintenance	0	122629.44	-	22629.44	-

Table 5 The benefits for the considered combinations

α	Policy	Number of maintenances	Total cost	$R_{tc}(50,15)$	Total operating cost	$R_{top}(50,15)$
5	No maintenance	0	144078	-	44078	-
	Under the policy	9	110358	0.766	10358	0.235
8	No maintenance	0	128062	-	28062	-
	Under the policy	6	109407	0.854	9407	0.335
10	No maintenance	0	122629	-	22629	-
	Under the policy	6	108468.1	0.885	8468.1	0.374
15	No maintenance	0	115295	-	15295	-
	Under the policy	4	107774.76	0.935	7774.76	0.508
20	No maintenance	0	111752	-	11752	-
	Under the policy	3	107906.29	0.966	7906.29	0.673

(3) While α gets smaller, the failure rate gets larger too. For $(C^p, M) = (50,15)$ in Table 5, the lease system under the policy saves much in both total cost and total operating cost while α gets smaller.

V. CONCLUSION

A maintenance policy with multiple choices of maintenance actions is proposed for the leased system. The system considered herein is under irregular inspection and the inspection times are dynamically updated based on the total number of failures to reflect the time-dependent deterioration. The policy is mainly to determine at each T_n the optimal maintenance action A_n^* and also the next T_{n+1}. Such policy can always get the minimum total expected cost over the leased periods by adjusting automatically both A_n^* and T_{n+1} based on the current system conditions. While we apply such policy to the leased photocopier system in practical, such system can save 11.5% in total cost and 63.6% in total operating cost.

Further improvements may go on along the lines: (1) take the time-dependent operation cost into consideration, and (2) considers the planning period *M* depends on C^p or $N_{T_n}^A$.

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