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微極液膜表面張力效應之直立外圓柱流的非線性液動穩定性分析

Surface Tension Effect on Nonlinear Waves in a Thin Micropolar Liquid Film Flow Down along a Vertical Cylinder

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摘要

本文利用長波微擾解所得到之廣義自由面運動方程式，探討直立圓柱面上微極流體薄膜流表面張力效應所表現之穩定性。首先以正模分析法來探討液膜的線性穩定性，進而得出線性中立穩定曲線、線性振幅增長率及線性波速。其次應用時間和空間之多重尺度法研究液膜的弱非線性穩定性質。研究結果顯示，當考慮表面張力時，流場將會產生亞臨界不穩定及超臨界穩定狀況。再者，微極流體之表面張力在穩定流場中具有消散擾動能量功能，因而減少對流之流動和表面波之波峰及波谷間的波動，即干擾後之振幅受到表面張力之影響而較易減少。而在流場不穩定時，表面張力增加擾動能量，故振幅增加更快。
關鍵詞：穩定性分析，表面張力

Abstract

The influence of surface tension effect on the nonlinear hydrodynamics stability of a thin micropolar liquid film flowing down along a vertical cylinder is investigated. A long-wave perturbation method is employed to solve generalized nonlinear kinematic equations at free film interface. The normal mode approach is first used to compute the linear stability solution for the film flow. The method of multiple scales is then used to obtain the weak nonlinear dynamics of the film flow for stability analysis. Results indicate that both subcritical instability and supercritical stability conditions possibly to occur in a micropolar film flow system with surface tension effect. It is shown that the flow stability in the stable states increases as surface tension value increases. However, the flow becomes somewhat unstable in unstable states as surface tension value increases.
Keywords: stability analysis, surface tension

I. INTRODUCTION

Benney [1] investigated the nonlinear evolution equation of free surface by using the method of small parameters. The solutions thus obtained can be used to predict nonlinear instability. However, the solutions cannot be used to predict supercritical stability since the influence of surface tension is not considered in the analysis of the small-parameter method. The effect of surface tension was realized by many researchers as one of the necessary conditions that will lead to the solution of supercritical stability. Lin [2], Nakaya [3] and Krishna and Lin [4] considered the significance of surface tension and treated it in terms of zeroth order terms in later studies. Pumir *et al.* [5] further included the effect of surface tension into the film flow model and solved for the solitary wave solutions. Hwang and Weng [6] showed that the conditions of both supercritical stability and subcritical instability are possibly to occur for a liquid film flow system. It is thus highly desirable to understand the underlying flow characteristics and associated time-dependent properties so that suitable conditions for homogeneous film growth can be developed for various industrial applications. In order to fully understand and characterize the stability conditions for various film flows, detailed flow analysis is of great importance.

Several researchers have already studied the hydrodynamic stability problems regarding the fluid films flowing down a vertical cylinder surface. Lin and Liu [7] com-

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pared their analytical solutions with the existing experimental results of falling flow film on a cylinder and creeping annular flow threads in viscous liquid. Krantz and Zollars [8] presented an asymptotic solution and pointed out that the effect of curvature on the stability of the film flow is indeed significant. They also showed that the curvature of the cylinder is indeed one of the important factors that intensify the instability of the film flow. This phenomenon is not found in the planar flow. Rosenau and Oron [9] derived an amplitude equation which describes the evolution of a disturbed free film surface traveling down an infinite vertical cylindrical column. The numerical modeling results indicated that both conditions of supercritical stability and subcritical instability are possible to occur for the film flow. The results also showed that the evolving waves may break at the instant that linearly unstable conditions are satisfied. Davalos-Orozco and Ruiz-Chavarria [10] investigated the linear stability of a fluid layer flowing down inside and outside of a rotating vertical cylinder. They pointed out that the centrifugal force could stabilize the film flow so as to counteract the destabilizing effect of surface tension. In the absence of rotation, the stability can still be found for some critical wave numbers. Hung *et al*. [11] investigated the weakly nonlinear stability analysis of a condensation film flowing down a vertical cylinder. They showed that supercritical stability in the linearly unstable region and subcritical instability in the linearly stable region can be co-exist. They also indicated that the lateral curvature of the cylinder has the destabilizing effect on the film flow stability.

A vast majority of studies on thin-film flow problems were devoted to the stability analysis of Newtonian fluids. The film flow of non-Newtonian fluids attracted less attention in the past. In recent years, the microstructure of fluid flows has emerged as a research subject of great interest to many researchers. A subclass of these fluids was named micropolar fluids by Eringen [12] who first proposed the theory of micropolar fluids. Micropolar fluids exhibit certain microscopic effects arising from the local structure and microrotations of the fluid elements. In application, the micropolar fluids may be used to model some man-made fluids, such as the polymeric fluids, animal blood, fluids with additives, and liquid crystals, etc. The extension of the theory of micropolar fluids to cover the thermal effect was developed by Eringen [13]. Liu [14] studied the flow stability of micropolar fluids and found that the initiation of instability was delayed due to the presence of microstructures in the fluid. Datta and Sastry [15] studied the instability of a horizontal micropolar fluid layer which was heated from below. They found that the plot of Rayleigh number versus wave number has two branches separating the zones of stability. Ahmadi [16] studied the same problem by employing a linear theory as well as an energy method. It was observed that the micropolar fluid layer heated from below is more stable as compared with the classical Newtonian fluid, and also found that, no subcritical instability region exists. Payne and Straughan [17] investigated the Benard problem for a thermo-micropolar fluid by the nonlinear energy stability method. They predicted the subcritical instability may possibly occur, but did not infer the existence of subcritical instabilities from their work. Later, Franchi and Straughan [18] established a nonlinear energy stability analysis for the convection of the thermo-micropolar fluid with temperature dependent viscosity. They showed that the critical Rayleigh number depends strongly on the changes of the interaction coefficient K, and indicated that the micropolar coefficient γ has very little influence on the convection threshold. Chang [19] employed the method of nonlinear analysis to study the stability of thin micropolar liquid films flowing down long a vertical moving plane. They also showed that the micropolar parameter $K(=\kappa/\mu)$ plays an important role in stabilizing the film flow.

The stability of a film flow is a research subject of great importance commonly needed in mechanical, chemical and nuclear engineering industries for various applications including the process of paint finishing, the process of laser cutting and heavy casting production processes. It is known that macroscopic instabilities can cause disastrous conditions to fluid flow. It is thus highly desirable to understand the underlying flow characteristics and associated time-dependent properties so that suitable conditions for homogeneous film growth can be developed for various industrial applications. The influence of the surface tension of micropolar liquid film on finite-amplitude equilibrium is studied and characterized mathematically. The sensitivity analysis of the surface tension is also carefully conducted. Several numerical examples are presented to verify the solutions and to demonstrate the effectiveness of the proposed modeling procedure.

II. GENERALIZED KINEMATIC EQUATIONS

Figure 1 shows the configuration of a thin micropolar liquid film flowing down along the outer surface of an infinite vertical cylinder. All physical properties are assumed to be constant. The principles of mass, momentum and angular momentum conservation for an axisymmetric isothermal incompressible micropolar flow configuration leads one to a set of system governing equations. Let u^* and w^* be the velocity components in r^* and z^* directions, respectively and N^* is the angular microrotation momentum. The governing equations can be expressed in terms of cylindrical coordinates $(r^*,\ z^*)$ as [13]

$$\frac{1}{r^*}\frac{\partial(r^*u^*)}{\partial r^*}+\frac{\partial w^*}{\partial z^*}=0 \tag{1}$$

$$\rho(\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial r^*}+w^*\frac{\partial u^*}{\partial z^*})$$
$$=-\frac{\partial p^*}{\partial r^*}+(\mu+\kappa)(\frac{\partial^2 u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}+\frac{\partial^2 u^*}{\partial z^{*2}}-\frac{u^*}{r^{*2}})$$
$$-\kappa\frac{\partial N^*}{\partial z^*} \tag{2}$$

$$\rho(\frac{\partial w^*}{\partial t^*}+u^*\frac{\partial w^*}{\partial r^*}+w^*\frac{\partial w^*}{\partial z^*}) =-\frac{\partial p^*}{\partial z^*}+(\mu+\kappa)(\frac{\partial^2 w^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial w^*}{\partial r^*}+\frac{\partial^2 w^*}{\partial z^{*2}}) +\kappa(\frac{\partial N^*}{\partial r^*}+\frac{N^*}{r^*})+\rho g \quad (3)$$

$$\rho J(\frac{\partial N^*}{\partial t^*}+u^*\frac{\partial N^*}{\partial r^*}+w^*\frac{\partial N^*}{\partial z^*}) =\gamma(\frac{\partial^2 N^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial N^*}{\partial r^*}+\frac{\partial^2 N^*}{\partial z^{*2}}-\frac{N^*}{r^{*2}}) +\kappa(\frac{\partial u^*}{\partial z^*}-\frac{\partial w^*}{\partial r^*}-2N^*) \quad (4)$$

where ρ is a constant density of the flow, p^* is the flow pressure, μ is the molecule fluid viscosity, J is the micro-inertial density, κ is the vortex viscosity and γ is the spin-gradient viscosity. The last term on the right-hand side of Equation (3) is the body force due to gravity.

The appropriate boundary conditions are:
At the cylinder surface ($r^*=R^*$):

$$u^*=0 \quad (5)$$

$$w^*=0 \quad (6)$$

$$N^*=\frac{1}{2}(\frac{\partial u^*}{\partial z^*}-\frac{\partial w^*}{\partial r^*}) \quad (7)$$

At free surface ($r^*=R^*+h^*$)

$$2(\frac{\partial u^*}{\partial r^*}-\frac{\partial w^*}{\partial z^*})\frac{\partial h^*}{\partial z^*}+(\frac{\partial u^*}{\partial z^*}+\frac{\partial w^*}{\partial r^*})[1-(\frac{\partial h^*}{\partial z^*})^2]=0 \quad (8)$$

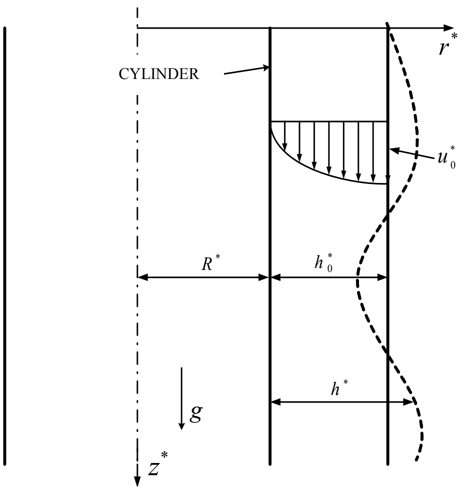

Fig. 1 Schematic diagram of a micropolar thin film flow traveling down along a vertical cylinder

$$p^*+2\mu[1+(\frac{\partial h^*}{\partial z^*})^2]^{-1}[(\frac{\partial u^*}{\partial z^*}+\frac{\partial w^*}{\partial r^*})\frac{\partial h^*}{\partial z^*}-\frac{\partial w^*}{\partial z^*}(\frac{\partial h^*}{\partial z^*})^2-\frac{\partial u^*}{\partial r^*}] +S^*\{\frac{\partial^2 h^*}{\partial z^{*2}}[1+(\frac{\partial h^*}{\partial z^*})^2]^{-3/2}-\frac{1}{r^*}[1+(\frac{\partial h^*}{\partial z^*})^2]^{-1/2}\}=p_a^* \quad (9)$$

$$N^*=0 \quad (10)$$

$$\frac{\partial h^*}{\partial t^*}+\frac{\partial h^*}{\partial z^*}w^*-u^*=0 \quad (11)$$

where h^* is the local film thickness. The boundary conditions at the interface, Equation (8) and Equation (9), are the balance of tangential and normal stresses [20]. In Equation (9), p_a^* is the atmosphere pressure, and S^* is the surface tension. The discussion of boundary conditions for the angular microrotation momentum N^* at solid wall and at free surface, shown in Equation (7) and Equation (10), can be found in Datta and Sastry [15] and Ahmadi [16]. Equation (11) is the free surface kinetic equation. The variable that is associated with a superscript "*" stands for a dimensional quantity. By introducing the stream function, φ^*, into dimensional velocity components, they become

$$u^*=\frac{1}{r^*}\frac{\partial \varphi^*}{\partial z^*},\quad w^*=-\frac{1}{r^*}\frac{\partial \varphi^*}{\partial r^*} \quad (12)$$

The dimensionless quantities can also be defined and given as

$$r=\frac{r^*}{h_0^*},\ z=\frac{\alpha z^*}{h_0^*},\ t=\frac{\alpha u_0^* t^*}{h_0^*},\ h=\frac{h^*}{h_0^*},\ \varphi=\frac{\varphi^*}{u_0^* h_0^{*2}},$$
$$p=\frac{p^*-p_a^*}{\rho u_0^{*2}},\ \mathrm{Re}=\frac{u_0^* h_0^*}{\nu},\ S=(\frac{S^{*3}}{2^4\rho^3\nu^4 g})^{1/3},\ N=\frac{N^* h_0^*}{u_0^*},$$
$$\Lambda=\frac{\gamma}{\mu J},\ K=\frac{k}{\mu},\ \Delta=\frac{h_0^{*2}}{J},\ \alpha=\frac{2\pi h_0^*}{\lambda} \quad (13)$$

where Λ, K and Δ are the dimensionless micropolar parameters, Re is the Reynolds number, R is the dimensionless radius of the cylinder, λ is the perturbed wave length, and α is the dimensionless wave number. h_0^* is the film thickness of local base flow and u_0^* is the reference velocity which can be expressed as [11]

$$u_0^*=\frac{g h_0^{*2}}{4\nu\Gamma} \quad (14)$$

where ν is the fluid kinematic viscosity, and

$$\Gamma=[2(1+R)^2\ln(\frac{1+R}{R})-(1+2R)]^{-1} \quad (15)$$

Thus, the non-dimensional governing equations and the associated boundary conditions can now be given as

$$p_r=\alpha\mathrm{Re}^{-1}[(1+K)(r^{-1}\varphi_{rrz}-r^{-2}\varphi_{rz})-KN_z]+O(\alpha^2) \quad (16)$$

$$(1+K)r^{-1}(r(r^{-1}\varphi_r)_r)_r-Kr^{-1}(rN)_r=4\Gamma +\alpha\mathrm{Re}(-p_z+r^{-1}\varphi_{tr}+r^{-2}\varphi_z\varphi_{rr}-r^{-3}\varphi_z\varphi_r-r^{-2}\varphi_r\varphi_{rz})+O(\alpha^2) \quad (17)$$

$$\Lambda(r^{-1}(rN)_r)_r-2K\Delta N+K\Delta(r^{-1}\varphi_r)_r =\alpha\mathrm{Re}(N_t+r^{-1}\varphi_z N_r-r^{-1}\varphi_r N_z)+O(\alpha^2) \quad (18)$$

at the cylinder surface $(r = R)$

$$\varphi = \varphi_r = \varphi_z = 0 \tag{19}$$

$$N = 2^{-1}(r^{-1}\varphi_r)_r + O(\alpha^2) \tag{20}$$

at free surface $(r = R + h)$

$$(r^{-1}\varphi_r)_r = 0 + O(\alpha^2) \tag{21}$$

$$p = -2S\cdot \mathrm{Re}^{-5/3}(2\Gamma)^{1/3}(\alpha^2 h_{zz} - r^{-1}) + \alpha\{-2\,\mathrm{Re}^{-1} [(r^{-2}\varphi_r - r^{-1}\varphi_{rr})h_z + r^{-2}\varphi_z - r^{-1}\varphi_{rz}]\} + O(\alpha^2) \tag{22}$$

$$N = 0 \tag{23}$$

$$h_t - r^{-1}(\varphi_r h_z + \varphi_z) = 0 \tag{24}$$

Subscripts of r, z, rr, zz and rz are used to represent various partial derivatives of associated underlying variables.

Since the long wave length modes (i.e. small wave number α) may introduce flow instability to meet our analysis objectives, the dimensionless stream function φ and pressure p are, therefore, expanded here in terms of some small wave number α as

$$\varphi = \varphi_0 + \alpha\varphi_1 + O(\alpha^2) \tag{25}$$

$$p = p_0 + \alpha p_1 + O(\alpha^2) \tag{26}$$

$$N = N_0 + \alpha N_1 + O(\alpha^2) \tag{27}$$

By substituting the above three equations into Equations (16)-(24), the system governing equations can then be collected and solved order by order. In the physical and mathematical justification, the non-dimensional surface tension S is a large value, the term $\alpha^2 S$ can be treated as a quantity in zeroth order [11]. Franchi and Straughan [18] and Hung *et al.* [21] showed that the micropolar parameter $\Lambda(=\gamma/\mu j)$ has very little effect on the stability of the micropolar film. For simplifying the solved equations that gives no spin-gradient viscosity effect, neglecting the Λ term of Equation (18), one can obtain the solutions of stream function for the equations of both the zeroth and the first orders given as

$$\varphi_0 = \frac{2\Gamma}{K+2}[\frac{1}{4}(r^4 + R^4) - \frac{1}{2}R^2 r^2 + \frac{1}{2}q^2(r^2 - R^2) - q^2 r^2 \ln(\frac{r}{R})] \tag{28}$$

$$\varphi_1 = k_1 r^6 + k_2 r^4 + k_3 r^2 + k_4 \tag{29}$$

where

$$k_1 = \frac{1}{6(2+K)^3}\mathrm{Re}\,\Gamma^2 q h_z \tag{30}$$

$$k_2 = \frac{S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}}{4(2+K)}(\alpha^2 h_{zzz} + \frac{1}{q^2}h_z) + \frac{\mathrm{Re}\,\Gamma q h_{0t}}{(2+K)^2}(\frac{5}{4} - \ln\frac{r}{R}) + \frac{\mathrm{Re}\,\Gamma^2 q h_z}{(2+K)^3}\{R^2(\frac{3}{2} - 2\ln\frac{r}{R}) +q^2[-12 + 12\ln\frac{r}{R} - 4(\ln\frac{r}{R})^2] + \frac{1}{\Delta}(-\frac{9}{4} + \ln\frac{r}{R})\} \tag{31}$$

$$k_3 = \frac{S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}}{2(2+K)}(\alpha^2 h_{zzz} + \frac{1}{q^2}h_z)(q^2 - R^2 - 2q^2\ln\frac{r}{R}) + \frac{\mathrm{Re}\Gamma h_{0t} q}{(2+K)^2}[q^2(2\ln Q - 1)(2\ln\frac{r}{R} - 1) - 2R^2] + \frac{\mathrm{Re}\Gamma^2 q h_z}{(2+K)^3} \{-\frac{5}{2}R^4 + R^2 q^2[20 - 4\ln\frac{r}{R} + 4(\ln\frac{r}{R})^2] + q^4[-7 + 12\ln Q + 14\ln\frac{r}{R} - 8(\ln Q)^2 - 24\ln Q\cdot\ln\frac{r}{R} + 16(\ln Q)^2\ln\frac{r}{R}] + \frac{1}{\Delta}[R^2(5 - 2\ln\frac{r}{R}) + q^2[1 + 6(\ln\frac{r}{R})^2 + 8\ln Q - 16\ln\frac{r}{R}\ln Q]\} \tag{32}$$

$$k_4 = \frac{S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}}{4(2+K)}(\alpha^2 h_{zzz} + \frac{1}{q^2}h_z)(R^4 - 2R^2 q^2) + \frac{\mathrm{Re}\Gamma h_{0t} q}{(2+K)^2}[\frac{3}{4}R^4 + q^2 R^2(2\ln Q - 1)] + \frac{\mathrm{Re}\Gamma^2 q h_z}{(2+K)^3}\{\frac{5}{6}R^6 - 8R^4 q^2 + R^2 q^4[7 - 12\ln Q + 8(\ln Q)^2] + \frac{1}{\Delta}[-\frac{11}{4}R^4 - R^2 q^2(1 + 2\ln\frac{r}{R} + 8\ln Q)]\} \tag{33}$$

and

$$q = R + h \tag{34}$$

$$Q = \frac{R+h}{R} = \frac{q}{R} \tag{35}$$

$$h_{0t} = \frac{4\Gamma}{2+K}(q^2 - R^2 - 2q^2\ln Q)h_z \tag{36}$$

By substituting the zeroth order and the first order solutions, φ_0 and φ_1, into the dimensionless free surface kinematic equation of Equation (24), the generalized nonlinear kinematic equation is obtained and presented as

$$h_t + A(h)h_z + B(h)h_{zz} + C(h)h_{zzzz} + D(h)h_z^2 + E(h)h_z h_{zzz} = 0 \tag{37}$$

where

$$A(h) = \frac{4\Gamma}{K+2}(R^2 - q^2 + 2q^2\ln Q) \tag{38}$$

$$B(h) = \frac{1}{4(K+2)}\alpha S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}(4q\ln Q - 3q + 4\frac{R^2}{q} - \frac{R^4}{q^3}) + \frac{\alpha\mathrm{Re}\Gamma^2}{(K+2)^3}\{\frac{13}{6}R^6 + \frac{1}{2}R^4 q^2(-9 + 28\ln Q) + \frac{1}{6}q^6[59 - 120(\ln Q)^2 + 96(\ln Q)^3] + \frac{1}{2}R^2 q^4[-15 - 8(\ln q)^2 - 68\ln Q + 16\ln q\cdot\ln Q + 32(\ln Q)^2 + 8(\ln R)^2] + \frac{1}{\Delta}[\frac{11}{4}R^4 + q^2 R^2(-4 + 12\ln Q) + \frac{1}{4}q^4(5 - 36\ln Q + 40(\ln Q)^2]\} \tag{39}$$

$$C(h) = \frac{1}{4(K+2)}\alpha^3 S\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}(4R^2 q + 4q^3\ln Q - 3q^3 - \frac{R^4}{q}) \tag{40}$$

$$D(h)=\frac{1}{2(K+2)}\alpha S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}(4\ln Q+\frac{R^4}{q^4}-1)$$
$$+\frac{\alpha\mathrm{Re}\Gamma^2}{(K+2)^3}\{\frac{13}{6}\frac{R^6}{q}+q^5[\frac{413}{6}-40\ln Q-92(\ln Q)^2$$
$$+112(\ln Q)^3]+R^4q(\frac{1}{2}+42\ln Q)+R^2q^3[-\frac{143}{2}$$
$$-130\ln Q+100(\ln Q)^2]+\frac{1}{\Delta}[\frac{11}{4}\frac{R^4}{q}+qR^2(36\ln Q)$$
$$+\frac{q^3}{4}(-11-100\ln Q+200(\ln Q)^2]\} \quad (41)$$

$$E(h)=\frac{2}{K+2}\alpha^3 S\,\mathrm{Re}^{-2/3}(2\Gamma)^{1/3}(R^2-q^2+2q^2\ln Q) \quad (42)$$

In case of $K=0$ and $\Delta=\infty$, the fluid flow becomes a typical classical Newtonian film flow. In case of $R=\infty$, the result agrees exactly with the solution of plane flow.

III. STABILITY ANALYSIS

The variation of film thickness in the base flow is found very small, so it is reasonable to assume that the local dimensionless film thickness equals to one. The dimensionless film thickness when expressed in perturbed state can be expressed as

$$h(t,z)=1+\eta(t,z),\qquad \eta=O(\alpha) \quad (43)$$

where η is a perturbed quantity to the stationary film thickness. By inserting the above equation into Equation (37) and collecting all terms up to the order of η^3, the evolution equation of η is obtained and given as

$$\eta_t+A\eta_z+B\eta_{zz}+C\eta_{zzzz}+D\eta_z^2+E\eta_z\eta_{zzz}=-[(A'\eta+\frac{A''}{2}\eta^2)\eta_z$$
$$+(B'\eta+\frac{B''}{2}\eta^2)\eta_{zz}+(C'\eta+\frac{C''}{2}\eta^2)\eta_{zzzz}+(D+D'\eta)\eta_z^2$$
$$+(E+E'\eta)\eta_z\eta_{zzz}]+O(\eta^4) \quad (44)$$

The values of A, B, C, D, E and their derivatives are all evaluated at the dimensionless height, h=1, of the film flow.

1. Linear Stability Analysis

As the nonlinear terms in Equation (44) are neglected, the linearized equation is obtained and given as

$$\eta_t+A\eta_z+B\eta_{zz}+C\eta_{zzzz}=0 \quad (45)$$

In order to use the normal mode method for analysis, we assume that

$$\eta=a\exp[i(z-dt)]+c.c. \quad (46)$$

where a is the perturbation amplitude, and c.c. is the complex conjugate counterpart. The complex wave celerity, d, is given as

$$d=d_r+id_i=A+i(B-C) \quad (47)$$

where d_r is the linear wave speed, and d_i is the linear growth rate of the amplitudes. For $d_i>0$, the flow is in unstable linearly supercritical condition. For $d_i<0$, the flow is in stable linearly subcritical condition.

2. Nonlinear Stability Analysis

The method of multiple scales is used to analyze the stability of the nonlinear system. Several associated notions are defined and expressed as

$$\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2} \quad (48)$$

$$\frac{\partial}{\partial z}\to\frac{\partial}{\partial z}+\varepsilon\frac{\partial}{\partial z_1} \quad (49)$$

$$\eta(\varepsilon,z,z_1,t,t_1,t_2)=\varepsilon\eta_1+\varepsilon^2\eta_2+\varepsilon^3\eta_3 \quad (50)$$

where ε is a small perturbation parameter, $t_1=\varepsilon t$, $t_2=\varepsilon^2 t$, $z_1=\varepsilon z$. After substituting the above expressions into Equation (44) and performing expansion and rearrangement, the equation can be obtained as

$$(L_0+\varepsilon L_1+\varepsilon^2L_2)(\varepsilon\eta_1+\varepsilon^2\eta_2+\varepsilon^3\eta_3)=-\varepsilon^2N_2-\varepsilon^3N_3 \quad (51)$$

where

$$L_0=\frac{\partial}{\partial t}+A\frac{\partial}{\partial z}+B\frac{\partial^2}{\partial z^2}+C\frac{\partial^4}{\partial z^4} \quad (52)$$

$$L_1=\frac{\partial}{\partial t_1}+A\frac{\partial}{\partial z_1}+2B\frac{\partial}{\partial z}\frac{\partial}{\partial z_1}+4C\frac{\partial^3}{\partial z^3}\frac{\partial}{\partial z_1} \quad (53)$$

$$L_2=\frac{\partial}{\partial t_2}+B\frac{\partial^2}{\partial z_1^2}+6C\frac{\partial^2}{\partial z^2}\frac{\partial^2}{\partial z_1^2} \quad (54)$$

$$N_2=A'\eta_1\eta_{1z}+B'\eta_1\eta_{1zz}+C'\eta_1\eta_{1zzzz}+D\eta_{1z}^2+E\eta_{1z}\eta_{1zzz} \quad (55)$$

$$N_3=A'(\eta_1\eta_{2z}+\eta_{1z}\eta_2+\eta_1\eta_{1z_1})+B'(\eta_1\eta_{2zz}+2\eta_1\eta_{1zz_1}$$
$$+\eta_{1zz}\eta_2)+C'(\eta_1\eta_{2zzzz}+4\eta_1\eta_{1zzzz_1}+\eta_{1zzzz}\eta_2)+D(2\eta_{1z}\eta_{2z}$$
$$+2\eta_{1z}\eta_{1z_1})+E(\eta_{1z}\eta_{2zzz}+3\eta_{1z}\eta_{1zzz_1}+\eta_{1zzz}\eta_{2z}+\eta_{1zzz}\eta_{1z_1})$$
$$+\frac{1}{2}A''\eta_1^2\eta_{1z}+\frac{1}{2}B''\eta_1^2\eta_{1zz}+\frac{1}{2}C''\eta_1^2\eta_{1zzzz}+D'\eta_1\eta_{1z}^2$$
$$+E'\eta_1\eta_{1z}\eta_{1zzz} \quad (56)$$

Equation (51) can now be solved order by order. After collecting the terms of order of $O(\varepsilon)$ and solving for the resulting equation $L_0\eta_1=0$, the solution can be easily obtained as

$$\eta_1=a(z_1,t_1,t_2)\exp[i(z-d_rt)]+c.c. \quad (57)$$

After collecting terms and solving for the secular equation of order $O(\varepsilon^2)$, the solution of η_2 gives

$$\eta_2=ea^2\exp[2i(z-d_rt)]+c.c. \quad (58)$$

By substituting both η_1 and η_2 into the equation of order $O(\varepsilon^3)$, the resulting equation becomes

$$\frac{\partial a}{\partial t_2}+D_1\frac{\partial^2 a}{\partial z_1^2}-\varepsilon^{-2}d_ia+(E_1+iF_1)a^2\bar{a}=0 \quad (59)$$

where

$$e = e_r + ie_i = \frac{(B' - C' + D - E)}{16C - 4B} + i\frac{-A'}{16C - 4B} \quad (60)$$

$$D_1 = B - 6C \quad (61)$$

$$E_1 = (-5B' + 17C' + 4D - 10E)e_r - A' e_i + (-\frac{3}{2}B'' + \frac{3}{2}C'' + D' - E') \quad (62)$$

$$F_1 = (-5B' + 17C' + 4D - 10E)e_i + A' e_r + \frac{1}{2}A'' \quad (63)$$

The overhead bar appeared in the above expressions stands for the complex conjugate of the same variable. Here, Equation (59) is generally referred to as the Ginzburg-Landau equation [22]. It can be used to investigate the weak nonlinear behavior of the fluid film flow. The solution of the exponential form is assumed and given as

$$a = a_0 \exp[-ib(t_2)t_2] \quad (64)$$

By employing the condition of a filtered wave that gives no spatial modulation, neglecting the diffusion term, and substituting the assumed solution of Equation (64) into Equation (59), one can obtain

$$\frac{\partial a_0}{\partial t_2} = (\varepsilon^{-2} d_i - E_1 a_0^2) a_0 \quad (65)$$

$$\frac{\partial [b(t_2)t_2]}{\partial t_2} = F_1 a_0^2 \quad (66)$$

Of course, if E_1 becomes zero, the Equation (65) is reduced to a linear equation. The second term on the right-hand side of Equation (65) is induced by the effect of nonlinearity. It can either decelerate or accelerate the exponential growth of the linear disturbance based on the signs of d_i and E_1. Equation (66) can be used to modify the perturbed wave speed caused by infinitesimal disturbances appeared in the nonlinear system. In the linear unstable region $(d_i > 0)$, the condition for a supercritical stable region to exist is given as $E_1 > 0$. The threshold amplitude, εa_0, is given as

$$\varepsilon a_0 = \sqrt{\frac{d_i}{E_1}} \quad (67)$$

and the nonlinear wave speed is given as

$$Nc_r = d_r + \varepsilon^2 b = d_r + d_i (\frac{F_1}{E_1}) \quad (68)$$

On the other hand, in the linearly stable region $(d_i < 0)$, if $E_1 < 0$, the film flow presents the behavior of subcritical instability, and εa_0 is the threshold amplitude. The condition for a subcritical stable region to exist is given as $E_1 > 0$. Also, the condition for a neutral stability curve to exist is $E_1 = 0$. Based upon the discussion presented above, various characteristic states of the Landau equation can be summarized and presented in Table 1.

Table 1 Various states of the Landau equation

linearly stable (subcritical region) $d_i < 0$	subcritical instability $E_1 < 0$	$\varepsilon a_0 < (\frac{d_i}{E_1})^{\frac{1}{2}}$	$a_0 \to 0$	conditional stability
		$\varepsilon a_0 > (\frac{d_i}{E_1})^{\frac{1}{2}}$	$a_0 \uparrow$	subcritical explosive state
	subcritical (absolute) stability $E_1 > 0$	$a_0 \to 0$		
linearly unstable (supercritical region) $d_i > 0$	supercritical explosive state $E_1 < 0$	$a_0 \uparrow$		
	supercritical stability $E_1 > 0$	$\varepsilon a_0 \to (\frac{d_i}{E_1})^{\frac{1}{2}}$ $Nc_r \to d_r + d_i \frac{F_1}{E_1}$		

IV. NUMERICAL EXAMPLES

Figure 1 shows the schematic diagram of a micropolar liquid film traveling down along a vertical cylinder. Physical parameters that are selected for study include (1) Reynolds numbers ranging from 0 to 15, (2) the dimensionless perturbation wave numbers ranging from 0 to 0.12, (3) the values of micropolar parameter K is 0.5, (4) the value of micropolar parameter Δ is 10, (5) the values of dimensionless radius distance including 10 and 20, and (6) the values of dimensionless surface tension including 4000, 6000, and 8000. A numerical example is presented here to illustrate the effectiveness of the proposed modeling procedure in dealing with the surface tension effect of a thin micropolar fluid film flowing down along a vertical cylinder. In order to validate the result of analytical derivation, a finite-amplitude perturbation generator is employed to disturb the system for both linear and nonlinear stability analyses. Based on the modeling results, the condition for thin-film flow stability can now be expressed as a function of Reynolds number, Re, dimensionless perturbation wave number, α, and dimensionless surface tension, S. Some important conclusions are made. The modeling results are also used to compare with the analytical solutions given in this paper and some other conclusive results appeared in the literature.

1. Linear Stability Solutions

The neutral stability curve was obtained by computing the conditions of linear stability for a linear amplitude growth rate $d_i = 0$. The stability of flow field (α-Re plane) is separated into two different regions by the neutral curve. In the linearly stable subcritical region, the perturbed small waves decay as the perturbation time period increases. However, in the linearly unstable supercritical region, the perturbed small waves grow as the perturbation time period increases. Figure 2 shows the neutral stability curves of the micropolar film flow with different values on the dimensionless surface tension S. The results indicate that the area of linearly unstable region $(d_i > 0)$ becomes larger for a decreasing S. Figure 3 show the temporal film growth rate of micropolar fluid for S=4000, 6000 and 8000. The temporal film growth rate increases for $\alpha < \alpha_c$, and decreases for $\alpha > \alpha_c$ as the value of S increases. It is also shown that the temporal film growth rate increases for an increasing Re in all numerical computations. The results also indicate that the larger the value of dimensionless surface tension S is, the higher the stability in the stable region $(d_i < 0)$ of a liquid film should be.

2. Nonlinear Stability Solutions

As the perturbed wave grows to finite amplitude, the linear stability theory is no longer valid for accurate prediction of flow behavior. The theory of nonlinear stability should be used to study whether the disturbed wave amplitude in the linear stable region will become stable or unstable. The problem that subsequent nonlinear evolution on disturbance in the linear unstable region will develop to a new equilibrium state (supercritical stability) with a finite amplitude or a unstable situation is also studied. As mentioned before, a negative value of E_1 can cause the system to become unstable. Such a condition in the linear region is referred to as the sub-critical instability. In other words, if the amplitude of disturbances is greater than the threshold amplitude, the amplitude of disturbed wave will increase. This is contradictory to the result predicted by using a linear theory. As a matter of fact such a condition in the subcritical unstable region can in some cases cause the system to become explosive.

The hatched areas near the neutral stability curves in Figure 4 reveal that both the subcritical instability condition $(d_i < 0, E_1 < 0)$ and the explosive supercritical instability condition $(d_i > 0, E_1 < 0)$ are possible to occur for all values of S that are used in this study. The results also show that the neutral stability curves of $d_i = 0$ and $E_1 = 0$ are shifted downward as the values of S increase. Therefore, the area of shaded subcritical instability region increases and the area of shaded supercritical instability region decreases as the values of S increase.

Figure 5 shows the threshold amplitude in subcritical unstable region for various wave numbers with different S values at Re=10 and R=20. The results indicate that the threshold amplitude εa_0 becomes smaller as the value of dimensionless surface tension S increases. In such situations, the film flow will become unstable. That is to say, if the initial finite-amplitude disturbance is less than the threshold amplitude, the system will become conditionally stable. On the other hand, if the initial finite-amplitude disturbance is greater than the threshold amplitude, the system will become explosively unstable.

In the linearly unstable region, the linear amplification rate is positive, while the nonlinear amplification rate is negative. Therefore, a linear infinitesimal disturbance in

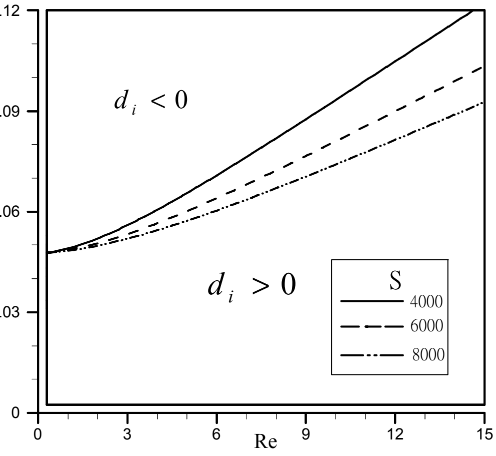

Fig. 2 Linear neutral stability curves for three different S values at R=20

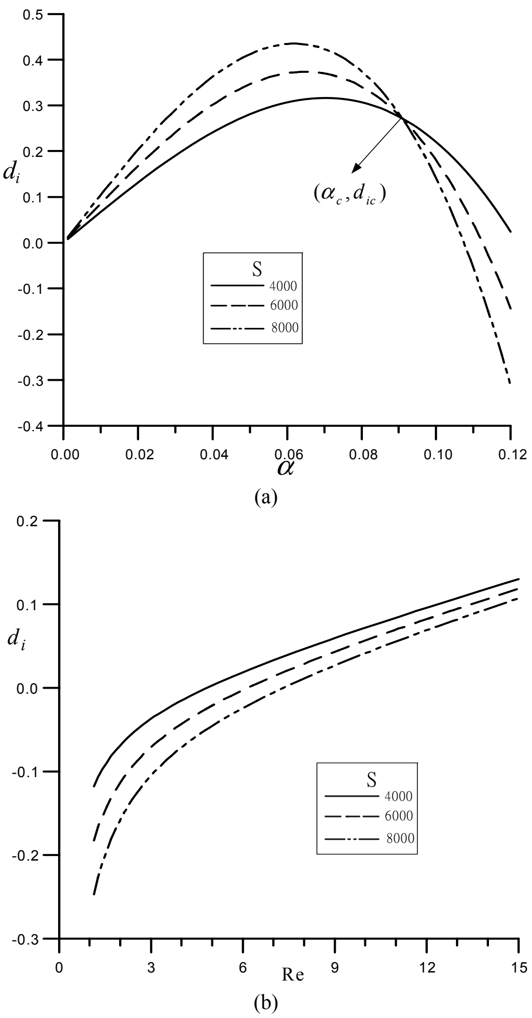

Fig. 3 Amplitude growth rate of disturbed waves in micropolar flows for three different S values at (a) Re=10 and R=10, (b) α=0.1 and R=10

the unstable region, instead of going infinite, will reach finite equilibrium amplitude as given in Equation (65). Figure 6 shows the threshold amplitude in the supercritical stable region for various wave numbers under different values of dimensionless surface tension S at Re=10 and R=20. It is found that the decrease of S will lower the threshold amplitude, and the flow will become relatively more stable.

The wave speed of Equation (47) predicted by using the linear theory is a constant value for all wave numbers and Reynolds number. However, the nonlinear wave speed, given by Equation (68), can be influenced by the wave number, Reynolds number, dimensionless surface tension S, and the radius of cylinder. The nonlinear wave speed is plotted in Figure 7 for various wave numbers and dimensionless surface tension S values at Re=10 and R=20. It is found that the nonlinear wave speed decreases as the value of S increases.

It is also noted that a cylinder with a smaller radius makes the flow relatively more unstable. This is due to the surface tension of the lateral curvature. In Equation (22),

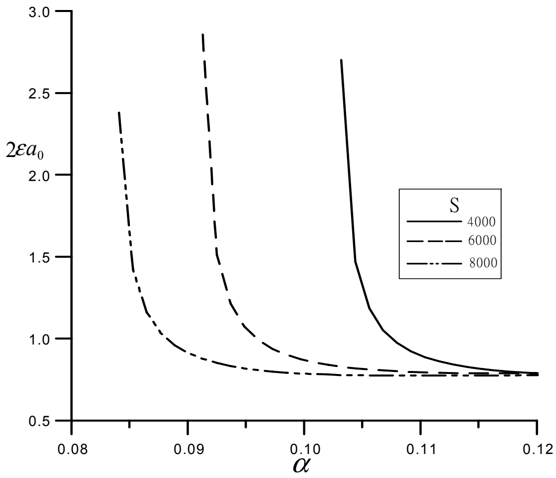

Fig. 5 Threshold amplitude in subcritical unstable region for three different S values at Re=10 and R=20

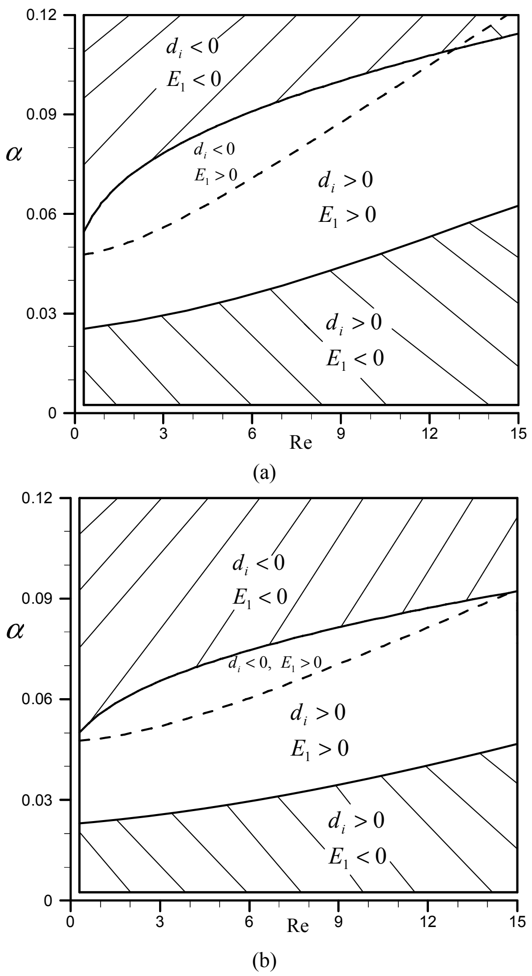

Fig. 4 Neutral stability curve of micropolar film flows for (a) S=4000 and R=20, (b) S=8000 and R=20

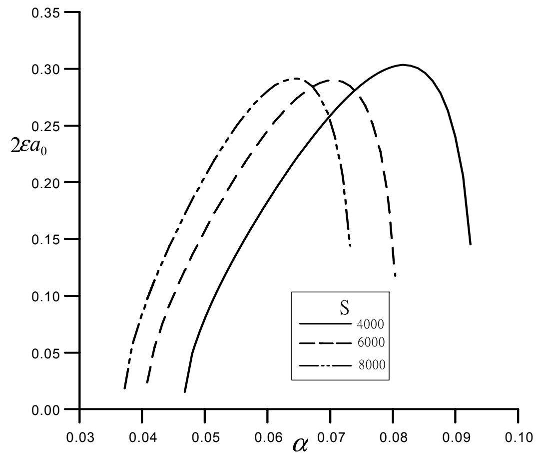

Fig. 6 Threshold amplitude in supercritical stable region for three different S values at Re=10 and R=20

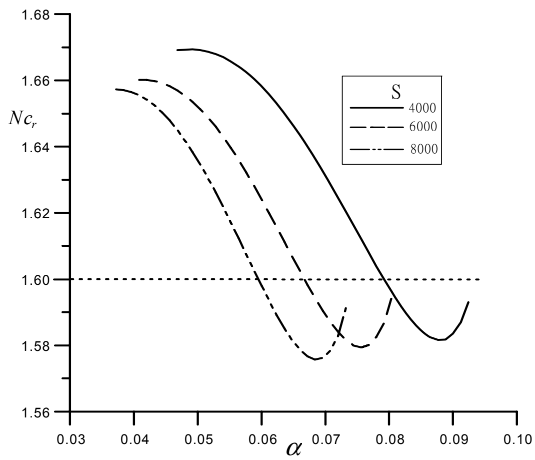

Fig. 7 Nonlinear wave speed in supercritical stable region for three different S values at Re=10 and R=20

the streamwise surface tension term, $S \cdot \mathrm{Re}^{-5/3}(2\Gamma)^{1/3}\alpha^2 h_{zz}$, is independent of the value of r. However, the lateral surface tension term, $S \cdot \mathrm{Re}^{-5/3}(2\Gamma)^{1/3} r^{-1}$, is inverse to the value of r. When the film flows down the outer surface of the cylinder with a smaller radius, the surface tension term of the lateral curvature will become larger. Therefore, it has a destabilizing effect. This destabilizing effect occurs because the radius of the trough of waves have a smaller value than that at the crest of the waves, and the surface tension will produce large capillary pressure at a smaller radius of curvature. This will induce the capillary pressure and force the fluid trough to move upward to the crest. Thus, the amplitude of the wave is increased.

As discussed above, it becomes apparent that the stability characteristic of a film flow traveling down along a vertical cylinder is significantly affected by the values of dimensionless surface tension S. That is to say, in nearly the complete working range, the degree of stability in stable states increases as the value of S increases. Because the effect of the microstructure in micropolar fluid will increase the effective viscosity, it can, therefore, reduce the convective motion of flow. Also, the degree of instability in unstable states increases as the value of S increases. By setting $R \to \infty$, the result becomes a solution for the plane flow problem. As compared to the modeling results given by Hung *et al.* [11], it is found that both solutions agree well with each other.

V. CONCLUDING REMARKS

The stability of a micropolar thin film flow traveling down along a vertical cylinder is thoroughly investigated in this paper by using the method of long wave perturbation. The generalized nonlinear kinematic equations of the free film surface near the wall are derived and numerically estimated to study the stability of flow field under different values of dimensionless surface tension. Based on the modeling results, several conclusions are as follows:

1. In the linear stability analysis, the neutral stability curve that separates the flow field into two different regions was first computed for a linear amplitude growth rate of $d_i = 0$. The modeling results indicate that the area of linearly unstable region becomes larger for a decreasing S. Furthermore, the flow becomes relatively stable if it is perturbed by short waves at a low Reynolds number and a larger surface tension in the stable region. It is also noted that the increasing value of S for smaller α and larger Re will increase the growth rate of temporal film in the unstable region.

2. In the nonlinear stability analysis, it is noted that the area of shaded subcritical instability region decreases as the value of S decreases. On the other hand, the area of shaded supercritical instability region increases with a decreasing S value. It is shown that the threshold amplitude εa_0 in the subcritical instability region decreases as the value of S increases. If the initial finite-amplitude disturbance is greater than the threshold amplitude value, the system will become explosively unstable. The threshold amplitude in the supercritical stability region increase with an increasing S value. The nonlinear wave speed in the supercritical stability region decrease with an increasing S value.

3. When the surface tension of a micropolor liquid is considered, it strongly affects the stability characteristic of a flow film. The simulation results indicate that the larger S will cause relatively stable in the stable region and will cause relatively unstable in the unstable region as traveling down along the vertical cylinder. In other words, the degree of stability in stable states increases as the value of S increases. Also, the degree of instability in unstable states increases as the value of S increases.

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