應用運動能量法於彈性平衡器之設計研究
Elastic Balancer Design Using Motion and Energy Synthesis

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摘要

機械系統之制動器在作動過程中，為了克服系統之重力因素，並在目前環境下達成工作目標，經常損失額外能量。本文基於運動及能量合成法，模擬機械系統的連桿設計；進而在傳統升降機升降過程中，設計適當的彈性平衡器來支撐制動器，並克服萬有引力作用而節省能量。彈性平衡器大都考量運動和能量，並由四連桿及彈簧來設計，通常需保持系統之總位能不變，若於系統中增加一較輕的彈性平衡器時，對系統之動態特性是不致有明顯改變。因此，本文藉由電腦程式模擬，設計並連接兩個彈性平衡器於制動器上；結果顯示，系統之最大力矩及最大超行量皆能降低百分之六十四。系統之彈性位能曲線常為不規則，本文提出以附加彈性平衡器之方式，不僅符合期望之彈性位能曲線，同時亦提供一套具體之四連桿及彈簧平衡器設計方案，以修正原系統之動態行為，並降低制動器輸出之力矩和力量。

關鍵詞：運動合成，能量合成，彈性平衡器，連桿設計

Abstract

The actuator of a mechanical/mechanism system often wastes a lot of energy to overcome the gravitational potential energy of the system in order to perform the essential work by the external circumstances. In this study, the motion and energy method are introduced and utilized to simulate the linkage design of a mechanical system. An appropriate elastic-balancer is additionally proposed to support the actuator in counteracting the gravitational force of a traditional elevator to save energy. The elastic-balancer composed of a four-bar linkage and a spring is considered through motion and energy synthesis. It also permits the total potential energy of the original system to remain constant. There is no significant influence to the dynamic characteristics when add a much lighter elastic-balancer to the original system. With the simulated results through the computerized program proposed, it is found that the two balancers connected to the actuator reduced 64 % in peak torque as well as the overshoot from the range. The desired elastic potential energy curve of a system is often irregular and not easy to fit. This paper not only fits the desired elastic potential energy curve to the summation curve of generated elastic potential energies by the balancer, but also provides a concrete design approach in synthesizing the four-bar linkage elastic balancers to modify the dynamics of the original system and therefore reduce actuator torque and force.

Keywords: motion synthesis, energy synthesis, elastic-balancer, linkage design

I. INTRODUCTION

The mechanism often maps out the motion and dynamic behavior for the desired work of a mechanical system; nevertheless, the waste of energy usually occurs.

For example, a robot is able to lift a mass of ten kilograms, but weights two hundred kilograms. The actuator usually has to lift the arm of the robot and the external mass, simultaneously. That is, we have to overcome the robot

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with two hundred kilograms in order to lift a mass of only ten kilograms. Some robots are even required to input energy continuously in supporting its weight during motionless, which commonly lose 80% to 90% of energy. With this viewpoint, it is necessary to design a force balancer in counteracting the gravitational potential energy of the system.

To either redistribute the mass of the original mechanical/mechanism system or add mass in the counter direction of the actuator [1, 2] is utilized in the traditional approaches to overcome the gravitational potential energy. These traditional approaches focused on the balance of the force and torque, and the dynamic characteristics of the original system that altered undesirably and simultaneously such as increased system response time were neglected. Therefore, designing a smaller and lighter balancer to improve the late defects becomes the critical issue in the field.

Gao and Song [3] have analyzed the foot force distribution of walking vehicles. Since the walking vehicles are very heavy and there is no balancer to help the leg actuator to overcome the gravitational forces; the normal force, longitudinal force, and lateral force applied to the legs are considerably large. Shin and Streit [4] have also designed a two-stage equilibrator to eliminate "geometric work" while allowing simultaneous dual action for each leg joint of a quadruped. Through a parallel five-bar linkage mechanism to transmit energy from one leg to another and several spring equilibrator elements, the system achieved balance. However, either the joint actuator torque or joint actuator work is nearly zero when the quadruped is operated at low speed and the leg kinematics energy is negligible. The dynamic simulation (walking under a speed of 2 mph) demonstrates that 90.3% of actuator torque can be reduced.

Properly adding the elastic potential energy elements to a system helps the actuators counteract the gravitational energy without changing the dynamic behaviors of the original system. The easiest way to balance a system is to use just only one compression/tension or torsion spring, where a spring is utilized to balance the weight of a slider-crank mechanism. Based on the consideration of the path from the path-tracer point, a spring cannot balance the weight of the whole system perfectly.

In other research, a spring with a four-bar linkage mechanism can be synthesized for motion generation with prescribed timing of the actuator [5, 6]. Surely, a six-bar linkage or an eight-bar linkage can also be used, but these are not discussed since there are many circuit problems and branches in multi-bar planar linkage analysis [7, 8]. Additionally, more sophisticated balancer by super imposing four-bar linkage spring balancer can be designed. Therefore; the four-bar linkage spring balancer design, the challenge in solving the planar linkage optimization problem, and the extensive adaptability are discussed in this study. This involves the testing of different optimization algorithms, modifying the parameters of the chosen optimization algorithm, providing a new methodology to produce a good control theory and combinatorial optimization [9, 10].

In this study, the position analysis of the four-bar linkage is analyzed by iterative method, and the velocity and acceleration of the four-bar linkage are analyzed by complex method. The motion synthesis requires that an entire body guided through a prescribed motion sequence. Additionally, the energy synthesis is introduced to generate a four-bar linkage with a spring element in order to modify the dynamics of the original system and therefore reduce actuator torque and force. Some applications are furthermore illustrated to introduce the various kinds of applications in motion synthesis and energy synthesis.

II. ENERGY SYNTHESIS

The principle of work and energy for a rigid body may be written as

$$T_1 + \sum U_{1-2} = T_2 \tag{1}$$

In Equation (1); T_1 and T_2 are the kinetic energy terms, and $\sum U_{1-2}$ represents the work done on the system.

When a body subjected to both gravitational and elastic energy forces, the total potential energy is expressed as a potential function V which can be described by the algebraic summation as shown below

$$V = V_g + V_e \tag{2}$$

In Equation (2); V_g and V_e are the gravitational and elastic potential energy, respectively.

The gravitational potential energy can be expressed as

$$V_g = W\ y_g \tag{3}$$

In Equation (3); W is the body's weight and y_g is the height of the body's center of gravity above or below a horizontal datum. The elastic potential energy which a spring with coefficient of elasticity k imparts to an attached body when the spring is stretched or compressed from an initial undeformed position ($s=0$) to a final position is derived as

$$V_e = +\frac{1}{2}\ k\ s^2 \tag{4}$$

It is shown in Equation (4) that the value of V_e is always positive.

The work done by a conservative force is measured by the difference as

$$\left(\sum U_{1-2}\right)_{cons.} = V_1 - V_2 \tag{5}$$

Introducing Equation (5) into Equation (1), the principle of work and energy for a rigid body can be rewritten as

$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{noncons.} = T_2 + V_2 \tag{6}$$

In Equation (6); $\left(\sum U_{1-2}\right)_{noncons.}$ represents the work of nonconservative forces of the mechanism system, such as friction, acting on the body. If this term is zero, then

$$T_1 + V_1 = T_2 + V_2 \tag{7}$$

Equation (7) is the equation of the conservation of mechanical energy for the body. It can be applied to a system of smooth, pin-connected rigid bodies. For a conservative system, if

$$V_1 = V_2 = \text{constant} \tag{8}$$

Then Equation (7) can be modified to

$$T_1 = T_2 = \text{constant} \tag{9}$$

It is observed from Equation (7), (8), and (9) that if the total potential energy of a system is manipulated constant, the kinetic energy of the system will still be constant. A cyclic system, such as crank-rocker or slider-crank four-bar linkage, will rotate continuously at a constant speed without the actuator (nonconservative force) energy input. Based on the above concept of the mechanism energy conservation, the four-bar elastic balancer design can thus be reasoned and discussed through Equation (7).

If the gravitational potential energy is known, by identifying the total potential energy as V_{total}, the desired elastic potential energy, $V_{e,des}\left(\theta_r\right)$, can be found from Equation (2) as

$$V_{e,des}\left(\theta_r\right) = V_{total} - V_g\left(\theta_r\right) \tag{10}$$

This can be shown in Figure 1; where V_{total} is an arbitrary choice and it is guaranteed that $V_{e,des}\left(\theta_r\right)$ is not negative, and θ_r is the input variable angle of the original system that can be regarded as the angle of the actuator or the angle of the driver of the four-bar linkage system.

In order to find a generated elastic potential energy curve to match the desired elastic potential energy curve, three prescribed points on the desired elastic potential energy curve shown in Figure 2 are selected.

Let $V_{e,des}\left(\theta_{ri}\right)$ be the change in desired elastic potential energy of the prescribed point, and s_i be the spring deformation which denotes the change in length from the undeformed spring length at any of the three precision point P_{ei}. From equation (4), it is then received that

$$s_i = \pm\sqrt{\frac{2V_{e,des}\left(\theta_{ri}\right)}{k}},\ i=1, 2, 3 \tag{11}$$

Here, one can choose either positive or negative for extension or compression state of the spring respectively. The spring length is then determined as

$$L_i = L_0 + s_i,\ i=1, 2, 3 \tag{12}$$

In Equation (12); L_0 is the initial undeformed length of spring which is a arbitrary choice. In Figure 3, the ground joint of spring is considered as the origin of coordinates, and the rotation angle of spring is ϕ_i when the original driver or actuator rotates to θ_{ri} (i =1, 2, 3).

Thus, the coordinate of the three-precision points are then expressed as

$$P_{ei} = \left(L_i \cos\phi_i, L_i \sin\phi_i\right),\ i=1, 2, 3 \tag{13}$$

After the three-precision points are found, the three-precision points synthesis is utilized to calculate the dyads of the four-bar linkage shown in Figure 4.

After the motion synthesis is done, one can analyze the energy-linkage and get all positions, velocities, and accelerations of each linkage in a cycle. The spring deformation is then given by

$$s\left(\theta_r\right) = \sqrt{\left(x_p - x_g\right)^2 + \left(y_p - y_g\right)^2} - L_0 \tag{14}$$

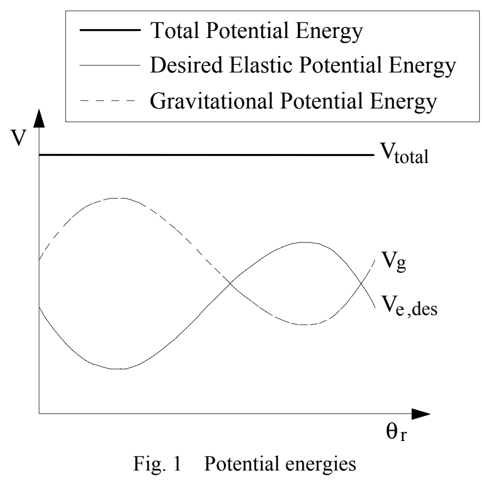

Fig. 1 Potential energies

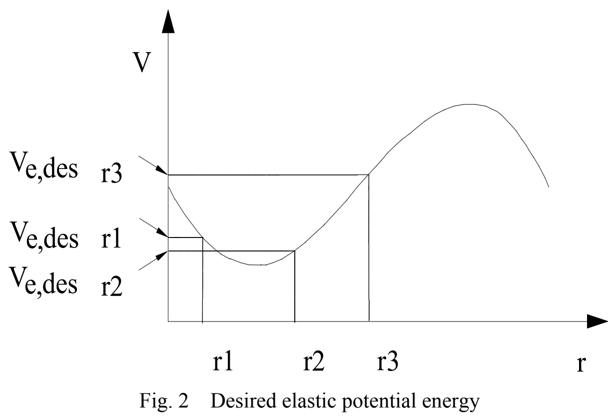

Fig. 2 Desired elastic potential energy

In Equation (14), (x_p, y_p) and (x_g, y_g) are the coordinates of the path-tracer point of energy-coupler and the ground joint of spring, respectively. The generated elastic potential energy from Equation (4) can then be obtained as

$$V_{e,gen}(\theta_r) = \frac{1}{2} ks^2(\theta_r) \tag{15}$$

When the path-tracer point of the energy-coupler moves to the three prescribed points, $V_{e,gen}(\theta_{ri})$ will be equal to $V_{e,des}(\theta_{ri})$. However, at the other points of the path, there may be some errors between $V_{e,gen}(\theta_r)$ and $V_{e,des}(\theta_r)$ (see Figure 5).

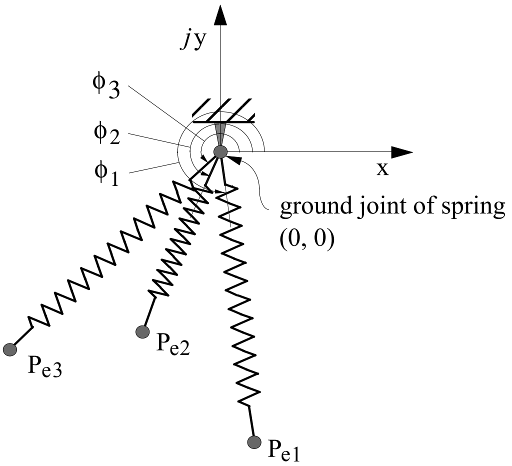

Fig. 3 Three states of spring

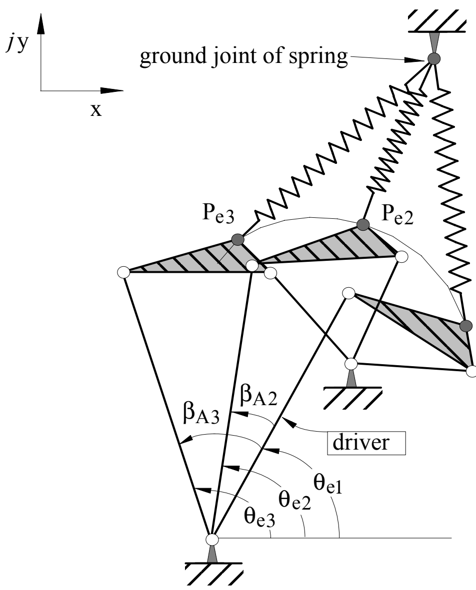

Fig. 4 Three states of the four-bar linkage spring balancer

Therefore, one should iterate the energy synthesis by using other values of the free choices for optimization. In this study of the energy synthesis, the variables of the three spring (V_{total}, k, L_0, θ_{r1}, θ_{r2}, θ_{r3}, β_{B2}, β_{B3}, α_2, α_3) can be chosen arbitrarily to be the state of extension or compression. Various values of these free choices are selected to not only find the different energy-linkage but also obtain the expected elastic potential energy curve based on the concrete design approach.

III. APPLICATIONS FOR ENERGY SYNTHESIS

A traditional elevator shown in Figure 6 is introduced as the application of this study. The balancing weight of 1.5 tons to balance the gravitational potential energy of the elevator is selected. This is observable from the example shown in Figure 6 where an elevator for reducing the driving force of the motor requires a balancing weight of 1.5 tons. It is shown that the most efficient condition delivers when the preload reaches around 1.5 tons.

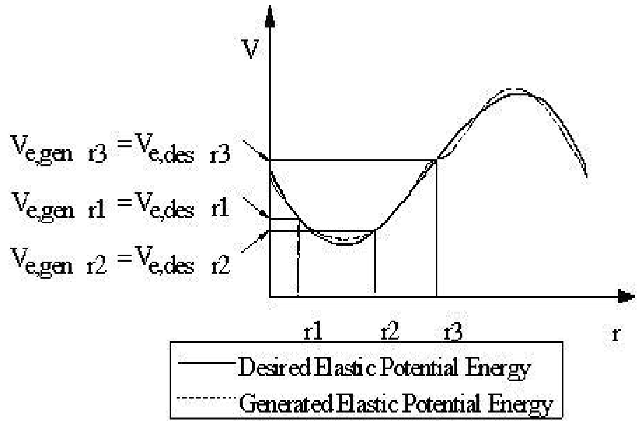

Fig. 5 Desired and generated elastic potential energies

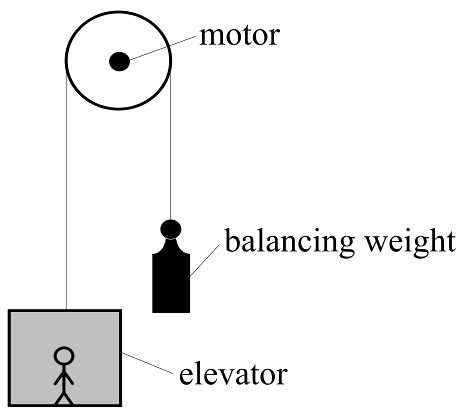

Fig. 6 Elevator with a balancing weight

In Figure 6, we assume that when the elevator of 1.5 tons weight is pulled from the bottom floor from *0 m* height to top floor of *15 m* height, and the motor should rotate *n* revolutions. The gravitational potential energy is calculated from a datum of the bottom floor. It is shown that the total potential energy is

$$1500\ kg * 9.8\ m/s^2 * 15\ m = 22050\ J$$

It is hardly possible to find a generated elastic potential energy curve in fitting such a desired elastic potential energy curve as shown in Figure 7. The rotating angle of the actuator can be restrained in *60* degrees by adding a gearbox of ratio *1:6n*. And, the potential energy curves are then changed as shown in Figure 8. It is more simple to fit such a desired elastic potential energy curve.

The Elastic potential energies for the elevator are shown in Figure 9 and the balancer is shown in Figure 10. It is noted from Figure 9 that the generated balancer has to rotate an angle of 22.07° when cooperated with the origin of the coordinate system since the energy synthesis is started from 22.07°. The simulated results of the elevator example are thus shown in Figures 11 through 12. From $W_{noncons.(\max)}$ =*220500 J* in Figure 8 and $W_{noncons.(\max)}$ =*254830.51-201942.08=52888.43 J* in Figure 11, it is obtained that the work of the maximun actuator effectiveness is 76 %. From the torque shown in Figure 12, the torque over the range is reduced even the peak torque is not reduced.

Using superposition perspectives in this example, it is shown that the desired elastic potential energy curve of the result in the elevator system cannot be fitted perfectly because of its irregularity or the limitation of arbitrary-chosen

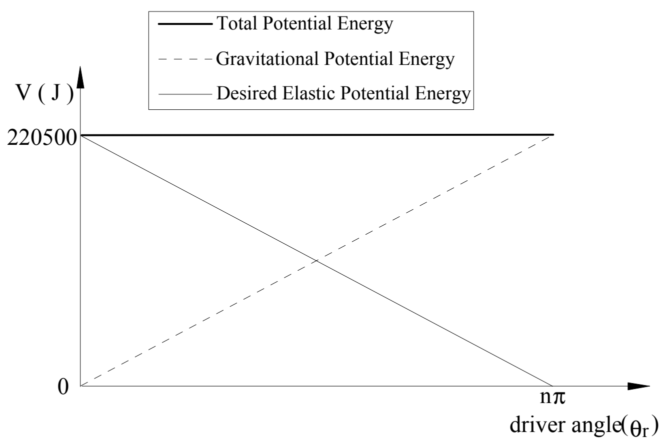

Fig. 7 Energies for elevator example

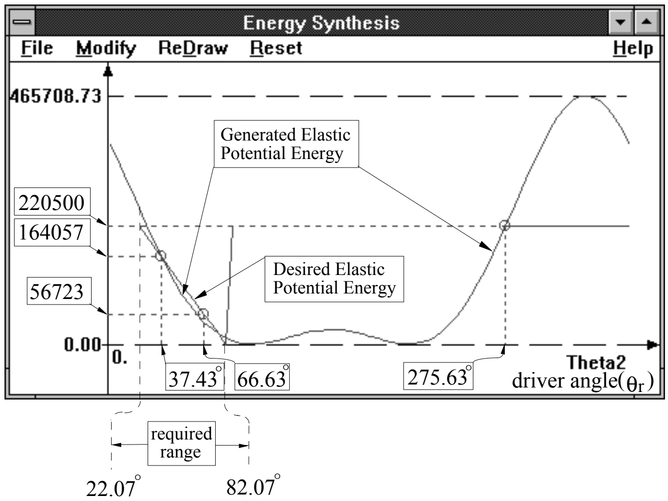

Fig. 9 Elastic potential energies for elevator example

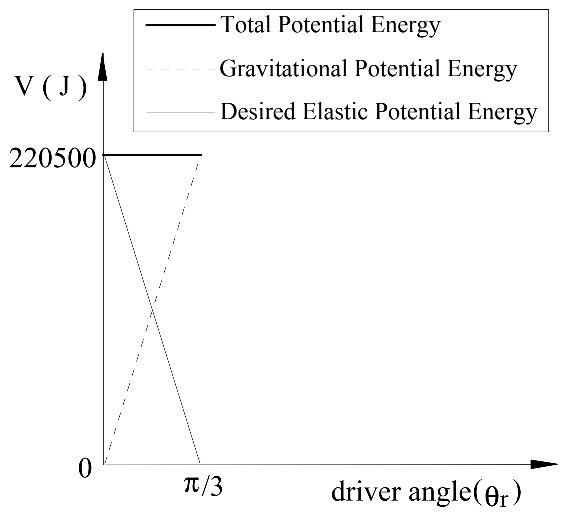

Fig. 8 Restrained Energies for elevator example

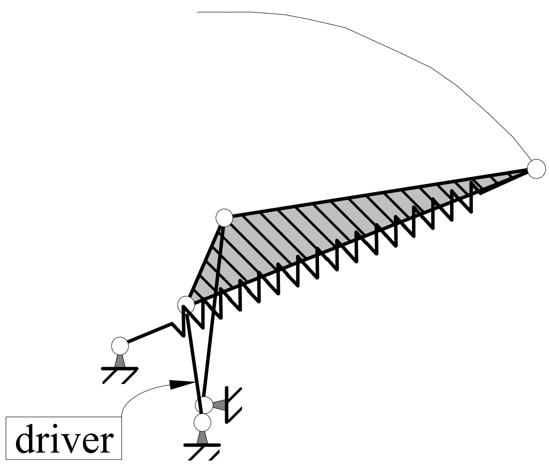

Fig. 10 Balancer for elevator example

parameters. Fitting the desired elastic potential energy curve is used to approach synthesizing many balancers as well as the summation curve of the generated elastic potential energy.

In Figure 13; the gravitational, desired and total potential energies of a system are shown. A spring with coefficient of elasticity *100 Nt/m* and initial undeformed length *0.8m* with unforced are selected. However, one cannot generate an elastic potential energy curve to fit this desired elastic potential energy curve perfectly because of some limitations. The most suitable generated elastic potential energy curve shown in Figure 13 is derived in this study.

In Figure 14, there still exists significant difference between these two elastic potential energy curves. Therefore, the second balancer is with desired elastic potential energy is calculated by the desired potential energy minus the generated elastic potential energy. Since there is no negative elastic potential energy and the minimum value

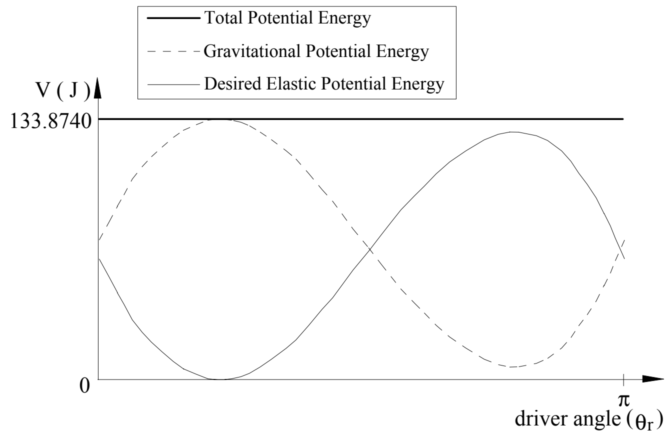

Fig. 13 Energies for superposition example

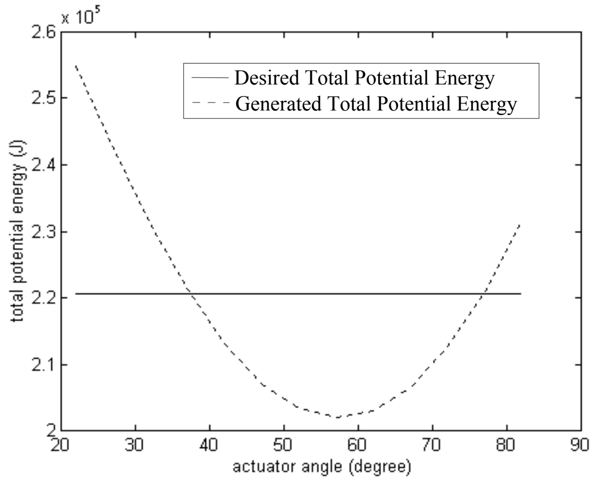

Fig. 11 Desired and generated total potential energies for the elevator

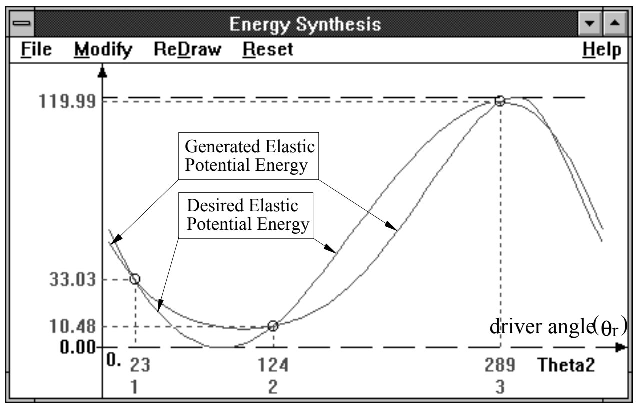

Fig. 14 Elastic potential energies of the first balancer

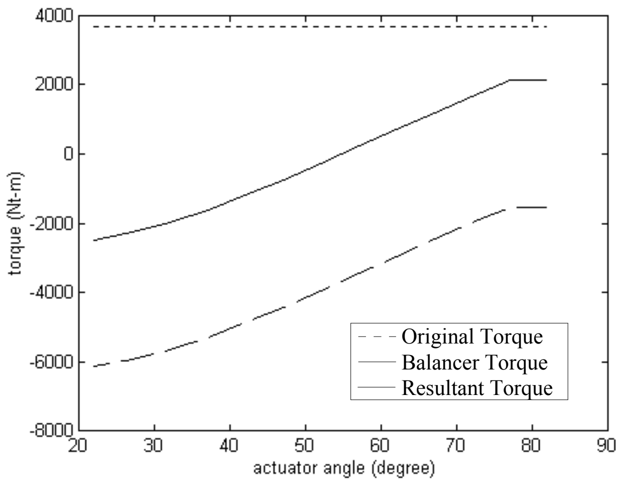

Fig. 12 Torque for the elevator example

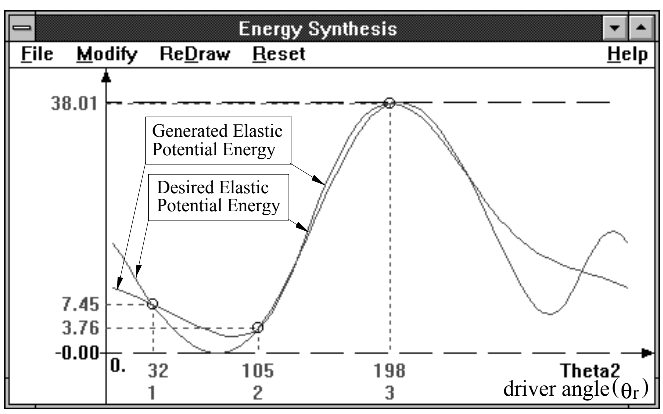

Fig. 15 Elastic potential energies of the second balancer

of the second desired elastic potential energy is -10.29J, we add a value of 10.29J to this energy for adjusting the minimum value to zero. Thus, the second desired and generated elastic potential energy curve is shown in Figure 15. The two balancers which are connected to the actuator of the original system are shown in Figure 16.

The results of the superposition example are shown from Figures 17 to 19. In Figure 17 and 18, the average last desired elastic potential energy and the average last total potential energy are added by approximately *10 J* due to the second balancer. As the result in Figure 18, the reduction in maximum work is about 68 % by the first balancer and about 87% after adding the second balancer. In Figure 19, an average of 64 % reduction in peak torque is realized and the torque over the range is also reduced. With the simulated results, this study shows good agreements to the desired objective.

IV. CONCLUSIONS

In the aspect of energy synthesis in this study, the design is demonstrated by adding a suitable elastic potential energy to let the total potential energy of the original system maintain constant. This surely helps the actuator counteracting the gravitational force of the original system by

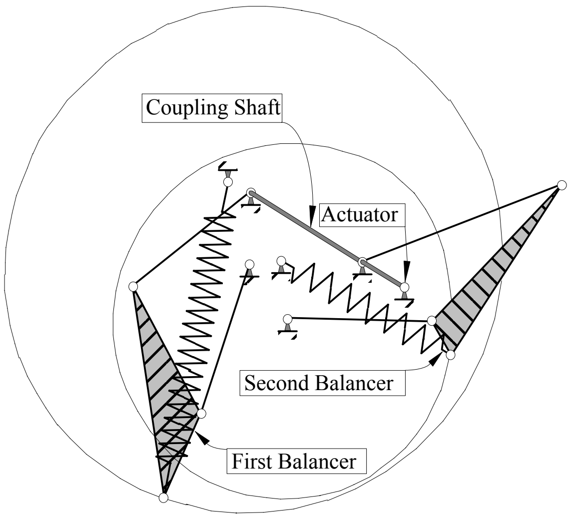

Fig. 16 Balancer for superposition example

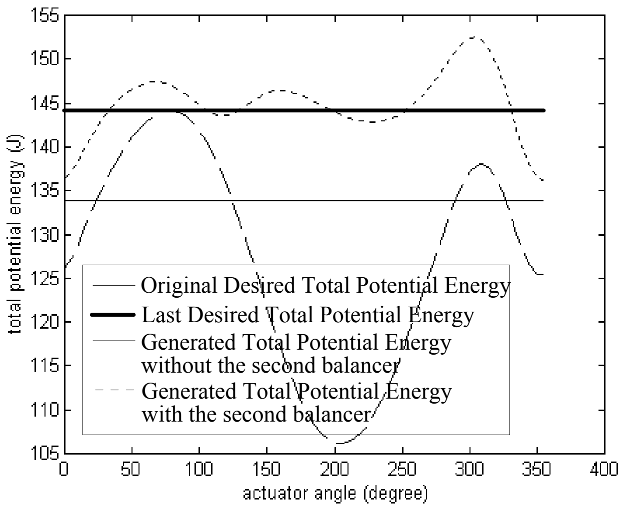

Fig. 18 Total potential energies for superposition

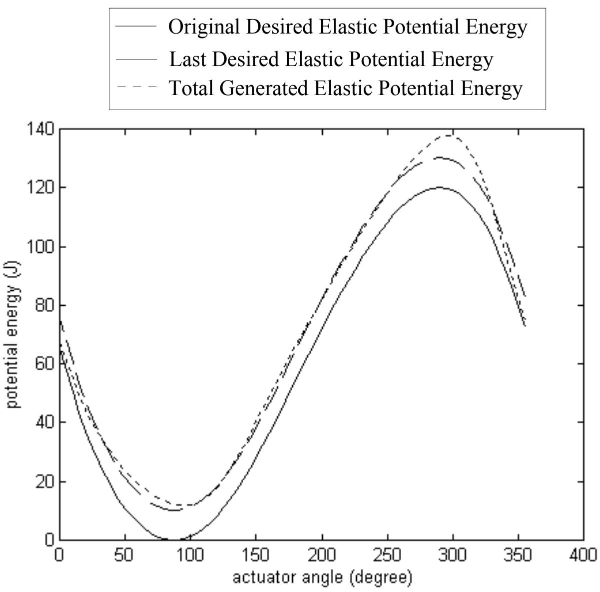

Fig. 17 Elastic potential energies for superposition

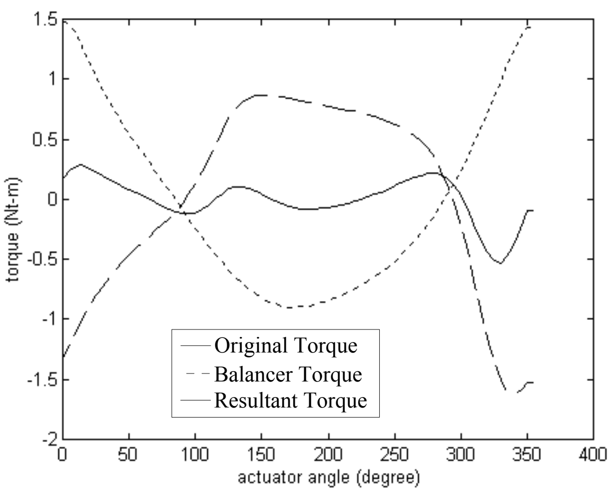

Fig. 19 Torque for superposition example

itself. The proposed design scheme of the mechanism for synthesizing a four-bar linkage with a spring element generates the elastic potential energy curve to fit the desired elastic potential energy curve.

Additionally, the irregularity of the desired elastic potential energy curve is presented hard to fit through the elevator example. Moreover, a gearbox is added to the actuator in reducing the range over the fitted area. The approach to synthesize many four-bar linkage elastic balancers in cooperating with the original system is furthermore made for the summation curve of the generated elastic potential energies of the many balancers to fit the desired elastic potential energy curve. This paper not only fits the desired elastic potential energy curve to the summation curve of generated elastic potential energies by the balancer, but also provides a concrete design approach in synthesizing the four-bar linkage elastic balancers to modify the dynamics of the original system and therefore reduce actuator torque and force.

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