

# 類神經網路應用於橋樑結構模態識別 Modal Identification of a Highway Bridge Using Artificial Neural Networks

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## 摘要

橋樑結構之動力特性，是研究橋樑動力行為與損壞評估的重要參數，而系統識別技巧則是探求結構特性最佳方法。本文主要是提出一個利用類神經網路運算的系統識別方法。由於類神經網路是一種非傳統的運算方法，具有容錯性及可訓練性，對於線性或非線性問題之處理、預測與識別，皆具有相當效率與精度，因此，本文將運用類神經網路訓練由現地動力試驗所得之結構自由振動反應，再經由神經網路之映射函數識別出結構之頻率、阻尼比與振態形狀等。本文將以三跨預力連續橋樑為例，利用現地衝擊試驗獲得橋樑自由振動反應，再經由本文方法求得動力特性參數，由分析結果可知，本文識別方法相當精準。

**關鍵詞：**模態識別，高速公路橋樑，類神經網路

## Abstract

The present proposes a neural network-based method for determining the dynamic characteristic parameters of structures from free vibration data. The method employs the observed dynamic responses to train an appropriate neural network. Conventional back-propagation is used to train the artificial neural network. Then, the modal parameters are directly identified by applying weight matrices in the neural network approach. The modal parameters, including frequencies, damping and vibration shapes of a structure can then be determined accurately. The proposed identification procedure is used to determine the dynamic characteristics of a three-span highway bridge, whose characteristics are determined from its free vibration responses by conducting a field impact test.

**Keywords:** modal identification, highway bridge, artificial neural network

## I. INTRODUCTION

Over the last two decades, artificial neural networks (ANN) have gradually become powerful tools for pattern recognition, signal processing, control, and complex mapping, because of their excellent capacity for learning and high tolerance to partially inaccurate data. An artificial neural network model is a system with its input and output data based on biological nerves. The system may comprise numerous computational elements that operate in parallel and are arranged in patterns that resemble biological neural nets. A neural network is usually characterized by its computational elements, its network topology and the learning algorithm applied [1]. Of the various ANNs, including

the feedforward, the multilayered and the supervised neural network with the error backpropagation algorithm-the backpropagation network (BPN) is by far the most commonly used neural network learning model. Chen et al [2] explicated the identification of a discrete-time nonlinear system using neural network with a single hidden layer. Masri et al [3] investigated the potential of using neural network method to determine the internal forces of structure-unknown non-linear dynamic system. This study presents an efficient identification method, a neural network approach, to analyze data collected in a field measurement test. The technique is employed to determine the dynamic characteristics of a three-span highway bridge.

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## II. NEURAL NETWORK ALGORITHM

### 1. Backpropagation neural network

A BP networks, depicted in Fig. 1, consists of an input layer, one or more hidden layers and an output layer. Every node in each layer is connected to every node in the adjacent layer. Importantly, Hecht-Nielsen [4] proved that one hidden layer of neurons suffices to model any solution surface of practical interest. Therefore, this study addresses a network with only one hidden layer. Before an ANN can be applied, it must be trained using an existing training set of pairs of input-output elements. The training of a supervised neural network using a BP learning algorithm typically involves three stages. The first stage is to feed-forward the data. The computed output of the  $i$ th node in the output layer is defined as follows;

$$y_i = g\left(\sum_{j=1}^{N_h} w_{ij} g\left(\sum_{k=1}^{N_i} v_{jk} x_k + \theta_{vj}\right) + \theta_{wi}\right), \quad i = 1, 2, \dots, N_o \quad (1)$$

Where  $w_{ij}$  is the connective weight between nodes in the hidden layer and those in the output layer;  $v_{jk}$  represents the connective weight between the nodes in the input layer and those in the hidden layer;  $\theta_{wi}$  (or  $\theta_{vj}$ ) are bias terms that represent the threshold of the transfer function  $g$ , and  $x_k$  is the input of the  $k$ th node in the input layer. Terms  $N_i$ ,  $N_h$ , and  $N_o$  are the numbers of nodes in the input, hidden and output layers, respectively. The transfer function can be linear or nonlinear. This work applies a linear transfer function. Typically, the input of each node in the input layer is normalized to be between 1 and -1 in the training of ANN.

The second stage is error back-propagation through the neural network. A system error function can be used to monitor the performance of the network. The system error function is commonly regarded as a function of the desired (or measured) value and the computed value of each node

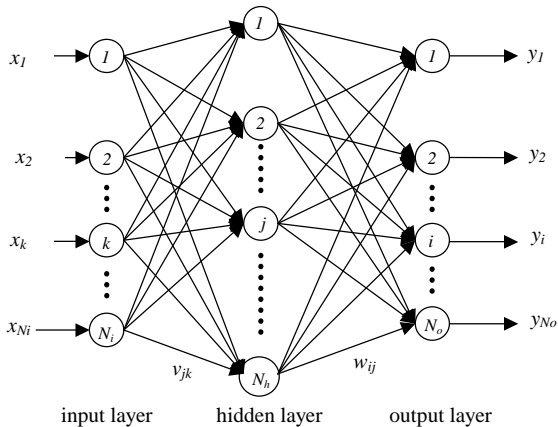


Fig. 1 A typical three-layer neural network

in the output layer. The final stage adjusts the weightings. A gradient descent method with a learning ratio in the standard BP algorithm is used to train the neural network. Normally, BP network learning models take an extended period to learn the network. Additionally, the convergence speed of a BP neural network depends on the learning ratio. For computational efficiency, the Marquardt-Levenberg algorithm [5] is applied herein to yield  $w_{ij}$ ,  $v_{jk}$ ,  $\theta_{wi}$  and  $\theta_{vj}$  by minimizing the system error function in this work.

### 2. Procedure for identifying dynamic parameters

The structural responses are obtained by determining  $x_i(t)$  from a numerical simulation or, experimentally, from a field vibration test. Therefore, a neural network to be established, as shown in Fig. 2, to predict  $x_i(t)$  from previous responses,  $x_i(t-j)$ , where  $i=1, 2, 3, \dots, n$ ;  $j=1, 2, \dots, m$ . Variables  $n$  and  $m$  represent the lags in the output and input, respectively.  $x_i(t-j)$  describes the observed historical responses of the  $i$ th measured degree of freedom, in relation to the basic at the  $(t-j)$ th time step. For a linear structural system, if the transfer function used is linear, Equation (1) can be easily applied to yield the neural network in Fig. 2, which can be mathematically described as,

$$\{Y\} = [W][V]\{X\} + ([W]\{\theta_w\} + \{\theta_o\}) \quad (2)$$

where  $\{Y\} = (x_1(t), x_2(t), \dots, x_n(t))^T$ ,

$$\{X\} =$$

$$(x_1(t-1) \ x_2(t-1) \ \dots \ x_n(t-1) \ x_1(t-2) \ x_2(t-2)$$

$$\dots \ x_n(t-2) \ \dots \ x_{n-1}(t-m) \ x_n(t-m))$$

The elements of  $[W]$  and  $[V]$  are  $w_{ij}$  and  $v_{ij}$ , respectively, and the elements of  $\{\theta_w\}$  and  $\{\theta_v\}$  are  $\theta_{wi}$  and  $\theta_{vi}$ . Carefully expanding equation (2) yields,

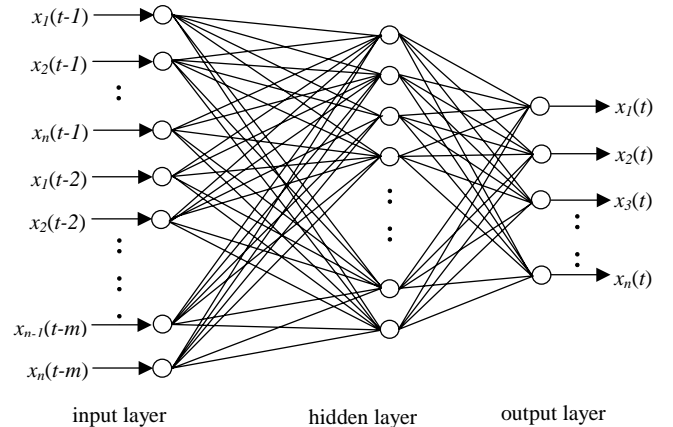


Fig. 2 Topology of a three-layer neural network

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{Bmatrix} = \sum_{j=1}^m \hat{\mathbf{W}}^{(j)} \begin{Bmatrix} x_1(t-j) \\ x_2(t-j) \\ \vdots \\ x_n(t-j) \end{Bmatrix} + \{C\} \quad (3)$$

where  $[\hat{\mathbf{W}}] = [W][V]$ ,  $\hat{\mathbf{W}} = [\hat{\mathbf{W}}^{(1)} \hat{\mathbf{W}}^{(2)} \dots \hat{\mathbf{W}}^{(m)}]$ ,  $\{C\} = [W]\{\theta_w\} + \{\theta_v\}$ . The elements in  $[W]$  and  $[V]$  are  $w_{ij}$  and  $v_{ij}$ , respectively, and the elements of  $\{\theta_w\}$  and  $\{\theta_v\}$  are  $\theta_{w_i}$  and  $\theta_{v_i}$ . It has to be mentioned that Eq. (3) is only valid for free vibration response.

Equation (3) is similar to the time series model, AR. The AR model equates the equations of motion. Hence, the dynamic characteristics of the structural system can be obtained from the coefficient matrices of AR. The modal parameters can be determined from the eigenvalues and eigenvectors of  $[G]$  by using the procedure of Huang [6] to establish the following matrix:

$$[G] = \begin{bmatrix} 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \mathbf{I} \\ \hat{\mathbf{W}}_1^{(m)} & \hat{\mathbf{W}}_1^{(m-1)} & \dots & \hat{\mathbf{W}}_1^{(2)} & \hat{\mathbf{W}}_1^{(1)} \end{bmatrix} \quad (4)$$

The eigenvectors of  $[G]$  correspond to the mode shapes of the structural system of interest in the form of state variables, whereas the eigenvalues of  $[G]$  relate to the natural frequencies and damping ratios. Let  $\lambda_k$  and  $\{\psi_k\}$  represent the  $k$ th eigenvalue and eigenvector of  $[G]$ , respectively. The eigenvalue,  $\lambda_k$ , is a complex number, and can thus be expressed as  $a_k + ib_k$ . The complex conjugates of  $\lambda_k$  and  $\{\psi_k\}$  are also an eigenvalue and eigenvector, respectively. The natural frequency and modal damping of the system, as in Eq. (4) are given by

$$\tilde{\beta}_k = \sqrt{\alpha_k^2 + \beta_k^2} \quad (5)$$

$$\tilde{\xi}_k = -\alpha_k / \tilde{\beta}_k \quad (6)$$

where  $\tilde{\beta}_k$  is the pseudo-undamped circular natural frequency;  $\tilde{\xi}_k$  is the modal damping ratio;

$$\beta_k = \frac{1}{\Delta t} \tan^{-1} \left( \frac{b_k}{a_k} \right) \quad (7)$$

$$\alpha_k = \frac{1}{2\Delta t} \ln(a_k^2 + b_k^2) \quad (8)$$

and  $1/\Delta t$  is the sampling rate of measurement.

The particular composition of  $[G]$  is such that its eigenvectors exhibit the following property:

$$\{\psi_k\} = (\{\psi_k\}_1^T, \lambda_k \{\psi_k\}_1^T, \lambda_k^2 \{\psi_k\}_1^T, \dots, \lambda_k^{m-1} \{\psi_k\}_1^T)^T \quad (9)$$

where  $\{\psi_k\}_1$  is the modal shape of the system that corresponds to the natural frequency,  $\tilde{\beta}_k$ .

Notably, the number of eigenvalues of  $[G]$  in Eq. (5) commonly exceeds the number of natural frequencies of the structural system. Accordingly, as well as real mechanical modes, extra spurious modes are generated from the constructed ANN. However, the eigenvalues and eigenvectors that correspond to real mechanical modes are observed regularly as  $m$  increases.

Hence, the responses measured in the structure in the field vibration test show that a neural network can be established, as indicated in Fig. 2. The modal parameters of such a structural system can be determined using the foregoing method. Besides, The validity of the proposed method is confirmed by the numerical studies of a shear-building model [7].

### III. COMPARISON OF THE IDENTIFIED RESULTS WITH FINITE ELEMENT RESULTS

The other tested bridge is a unit of the elevated highway bridge system in the northbound line of constructed along the No. 1 National Freeway, which crosses Taipei City in Taiwan, which is completed in 1996 and is called the Yuan-Shan bridge. As depicted in Fig. 3, the Yuan-Shan bridge is a continuous three-span bridge, which consists of pre-stressed concrete box-girders with various cross-sections. The total length of the bridge is 360 m, and the lengths for three spans are 95, 155 and 110 m. The bridge is about 27m high about the ground level. The total bridge is supported by rollers (identified as "M" for "movable" in Fig. 3) in the longitudinal direction but are constrained in the transverse and vertical directions at the two far ends. Meanwhile, it is rigidly supported by the two central columns (identified as "R" for "rigid" in Fig. 3).

#### 1. Data processing and identified results

Figure 4 plots a typical set of recorded data concerning the responses in the transverse direction at various locations, when the measurement segments were subjected to an impulsive force in that direction. The figures reveal that the free vibration response can indeed be generated by the proposed testing procedure. The impact responses to large impacts after 10 sec were used to train an ANN, to reduce the effect of noise and thus yield better identified results. Figure 2 presents the architecture of the ANN with  $n=7$ ,  $m=12$  and 16 nodes in the hidden layer. The velocity responses in each measurement channel in the transverse direction were used to train an ANN. Table 1 lists the natural frequencies and the modal damping ratios identified from the impulse test by the trained ANN, while Fig. 5 shows the modal shapes.

Table 1 indicates that a total of six transverse modes were identified. A comparison of the identified frequencies with those that correspond to the peak of the auto-spectra

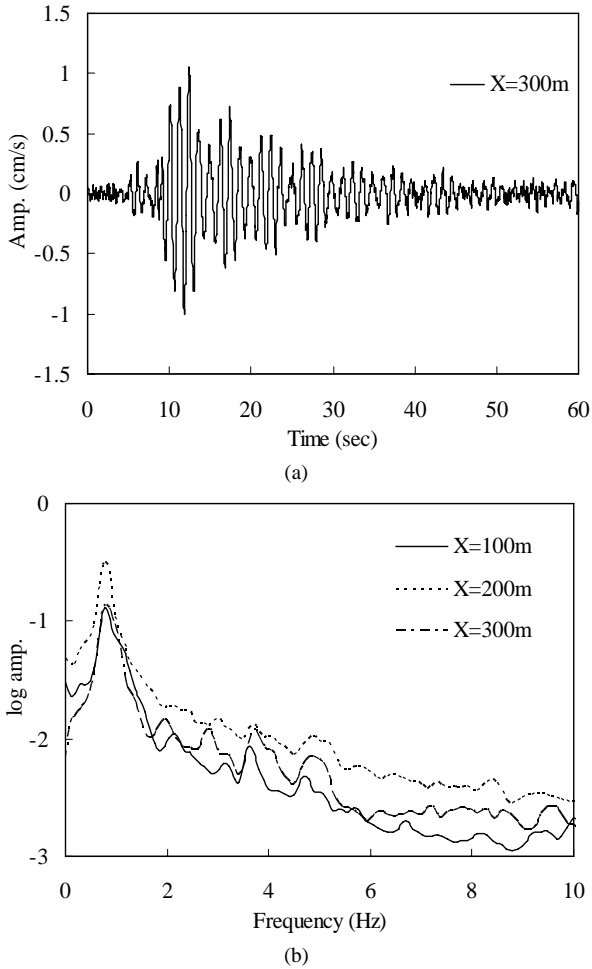


Fig. 4 A typical set of recorded data for the impact responses with the corresponding auto-spectra: (a) time history; (b) auto-spectra

Table 1 Identified natural frequencies and modal damping ratios

Mode	Impulse test		Ambient test		FEM
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)	
1	0.80	1.5	0.81	1.3	0.78
2	1.02	2.9	1.01	2.0	0.85
3	1.13	5.0	1.19	5.3	1.10
4	1.88	5.3	1.84	4.6	1.28
5	2.70	4.5	2.77	4.7	1.66
6	3.72	2.2	3.65	1.6	2.07
7	/	/	/	/	2.25
8	/	/	/	/	3.12
9	/	/	/	/	4.18

presented in Fig. 5 reveals the frequencies in the low modes agree with each other. However, the frequencies and damping ratios in higher modes in Table 1 cannot be directly identified from the auto-spectra. Table 1 also presents the results obtained in an ambient vibration test, using the random decrement system identification technique [8]. Table 1 reveals that the frequencies obtained in the impulse test agree very closely with those obtained in the ambient vibration test. Table 1 also shows that the consistency between the modal damping ratios identified from the two tests is not as strong as that between the natural frequencies, but is acceptable. The damping ratios identified obtained the tests are smaller than the design value of 5%, except in the fourth mode. These ratios do not increase with the natural frequency.

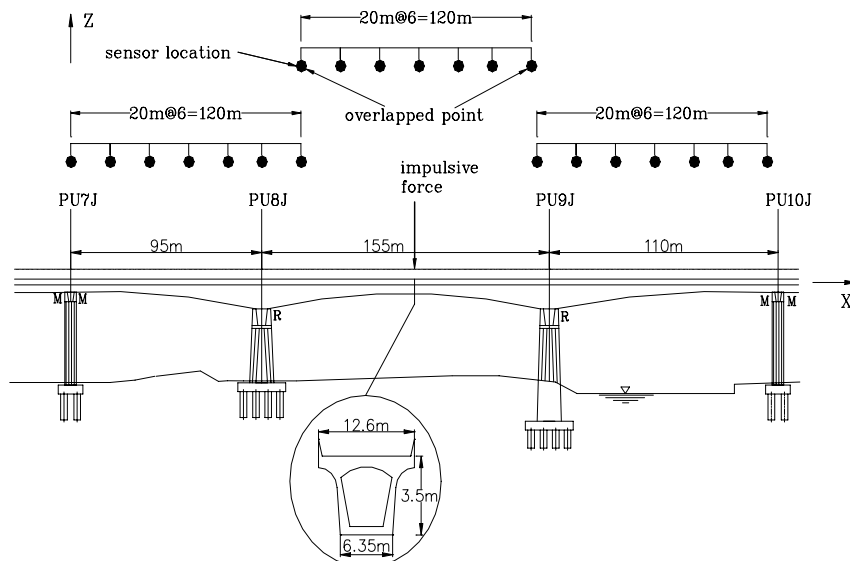


Fig. 3 The tested three-span highway bridge and the sensor layout

## 2. Finite Element Model

The finite element results shown in Table 1 and Fig. 4 were obtained by the commercial finite element package SAP90, using in particular the beam element that has six degrees of freedom at each node. In the finite element modeling, 89 beam elements were used for modeling the superstructure and 20 beam elements were used for the piers. These beam elements are assumed to be uniform cross-section. Designed material properties,  $E$  (Young's modulus) =  $2.806 \times 10^6$  kN/m<sup>2</sup> and  $G$  (shear modulus) =  $1.167 \times 10^6$  kN/m<sup>2</sup>, were used in this analysis. The soil-structure interaction was considered by using equivalent soil springs connected to the pile cap for each pier of the bridge [9]. Each of equivalent soil springs represents the stiffness of the rigid pile cap, assumed to be massless, associated with the movement in each of the three orthogonal directions or the rotation about each of the three orthogonal axes. Consequently, six frequency independent soil springs were established following the FHWA report by Lamé and Martin [10].

For comparison with measured results, the first ninth natural frequencies computed from the finite element analysis are listed in Table 1 as well, in which the dominant direction of vibration is determined from the maximum component of each eigenvector. In contrast,

only the modal shapes that are judged to be similar to the measured ones have been plotted in Fig. 4. The fourth analytical mode was excluded from Fig. 4, since its vibration shapes does not match any of the modes identified experimentally. The lack of consistency between the analytical and experimental results can be attributed mainly to the difference between the real structure and the finite element model, concerning the geometric and material properties, and boundary conditions. As a matter of fact, the results from the field tests serve as a useful basis for updating the finite element model.

## IV. CONCLUSIONS

This work uses artificial neural networks to determine the dynamic characteristics of a structural system from its dynamic responses. The approach is based on constructing a neural network to match the observed responses. The modal parameters of the structural system are directly estimated from the weighting matrices associated with the neural network. Then, the modal parameters are determined through matrix operations. The proposed procedure was applied to the free vibration responses by performing an impulse test on a three-span highway bridge.

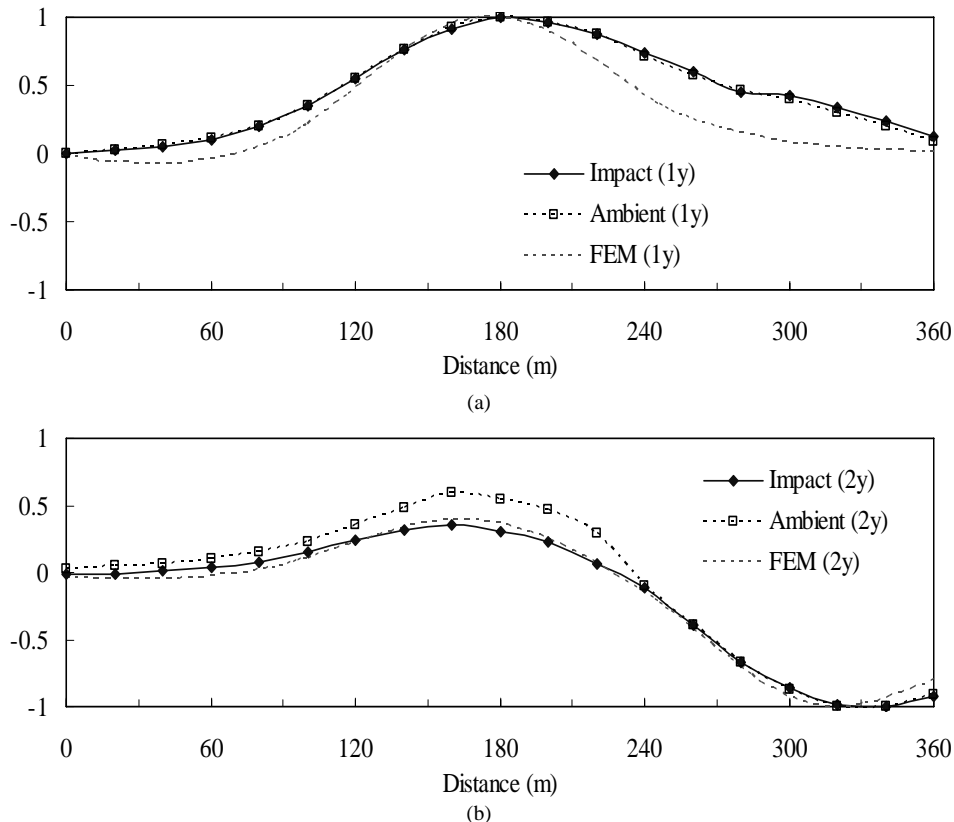
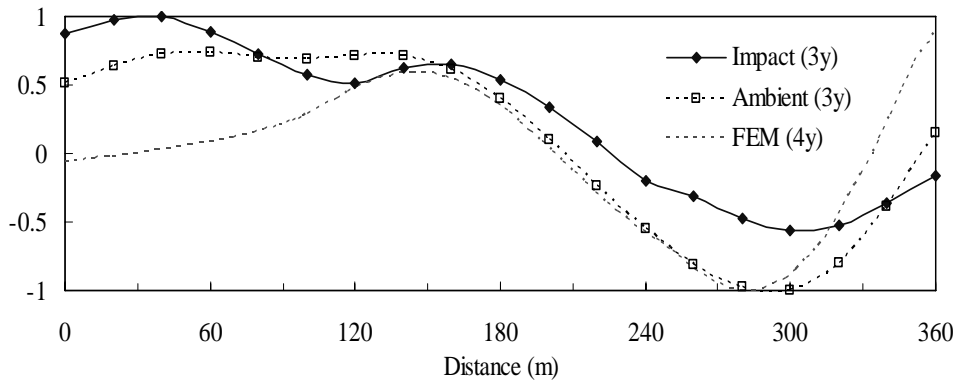
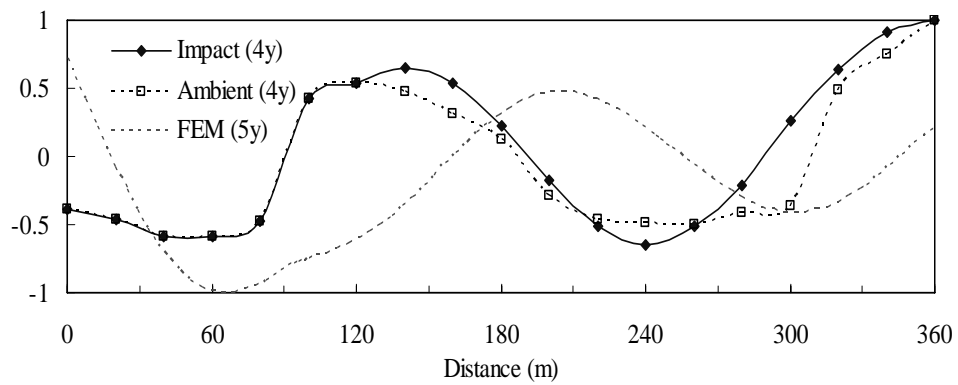


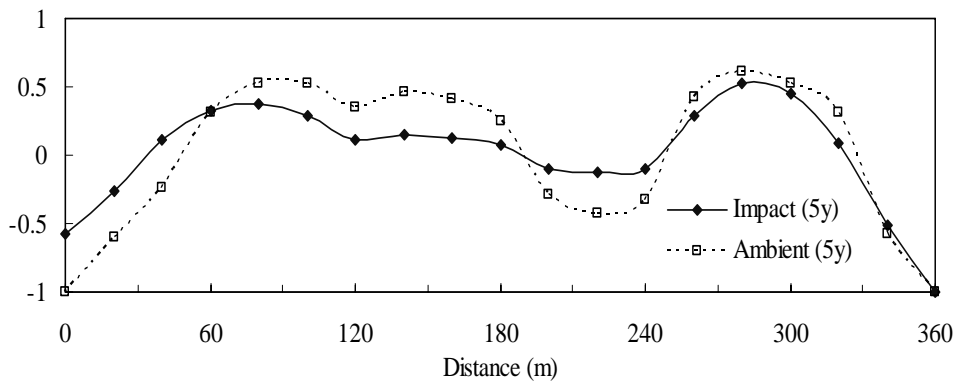
Fig. 5 Comparison of identified mode shapes: (a) mode1; (b) mode2; (c) mode3; (d) mode4; (e) mode5; (f) mode6



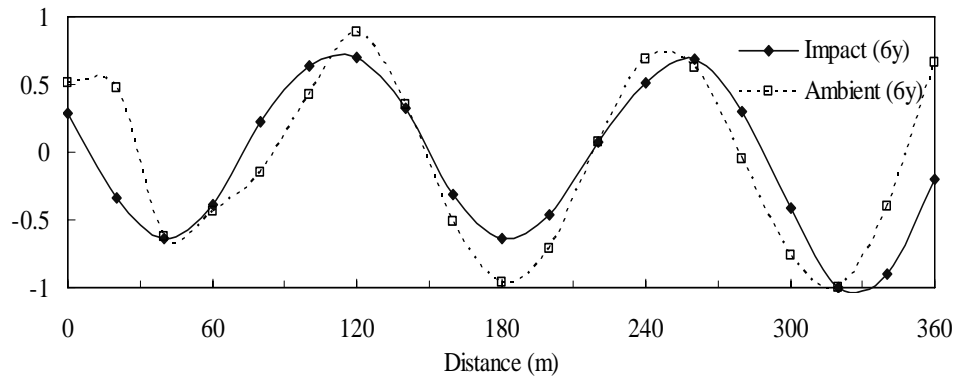
(c)



(d)



(e)



(f)

Fig. 5 Continued

The usefulness of the proposed method for estimating the dynamic characteristics of the bridge in the impulse test was confirmed by the excellent agreement between the obtained results and those in the ambient vibration test.

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